Transport via Resonances and Close Encounters for Dust and Spacecraft

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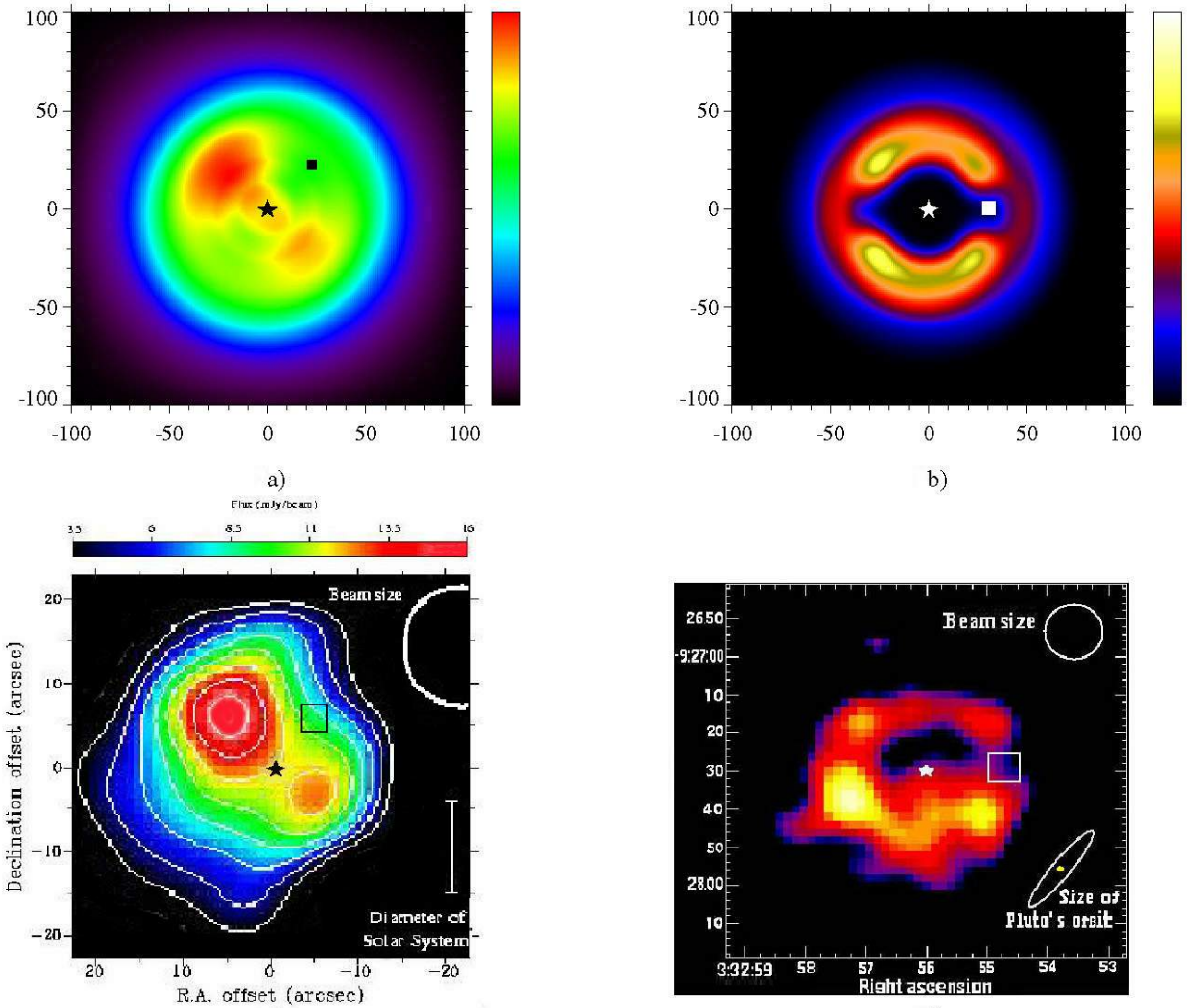
Introduction

- ***Resonances and close encounters play a key role in:***
  - ☐ Circumstellar dust disk evolution
  - ☐ Low energy spacecraft trajectories
- ***Current research importance***
  - ☐ Extrasolar planets may be detectable from their “signatures” in dust disks
  - ☐ Mission trajectories consuming little fuel can be designed
    - routes from Earth orbit to lunar orbit and beyond
    - a tour of Jupiter’s moons

Planet Detection

Circumstellar dust structures may reveal planets



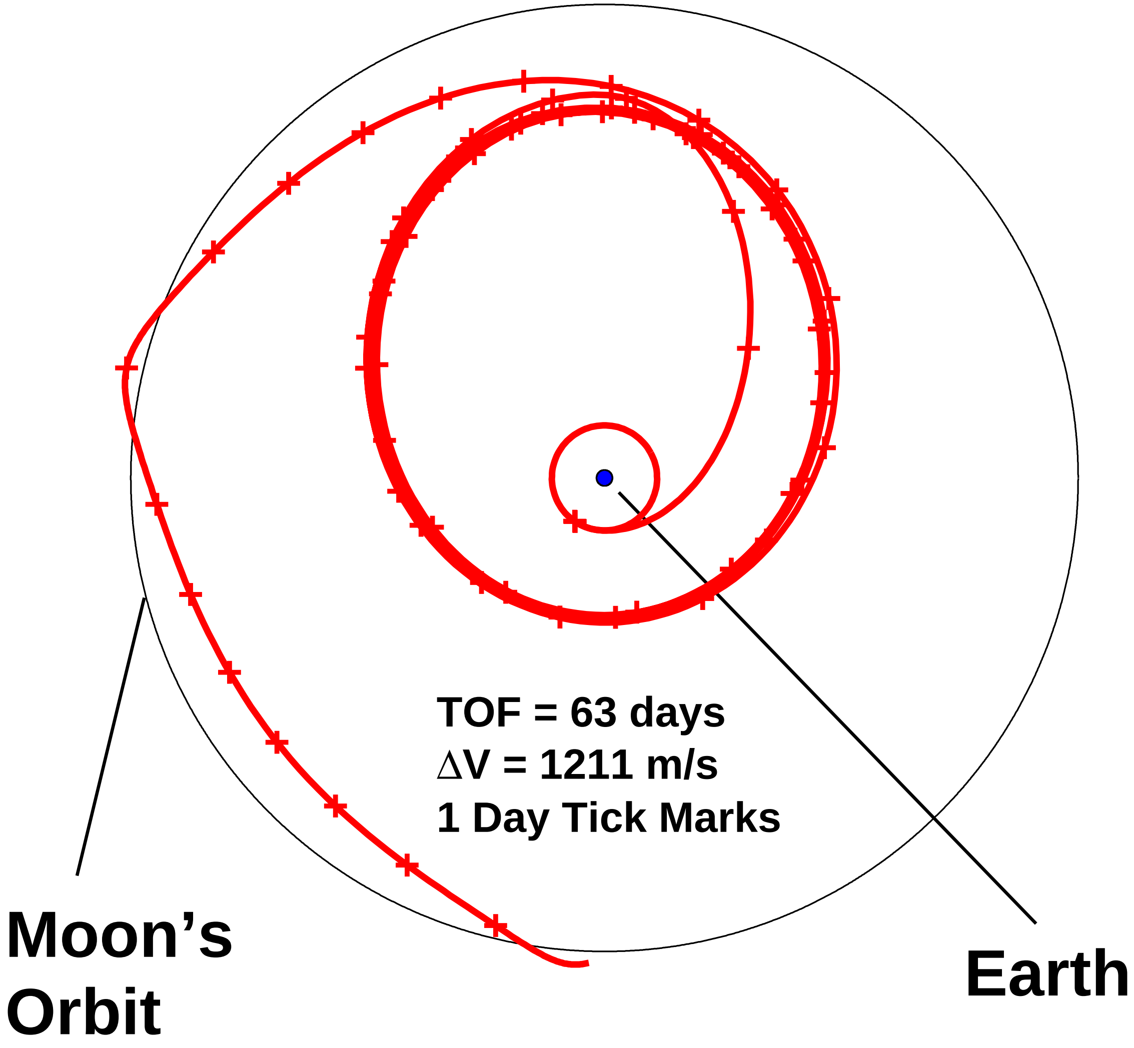
a) b) c) d)

Source: NASA, the George Mason University, and the Joint Astronomy Center (Hawaii)

Low Energy Transfers

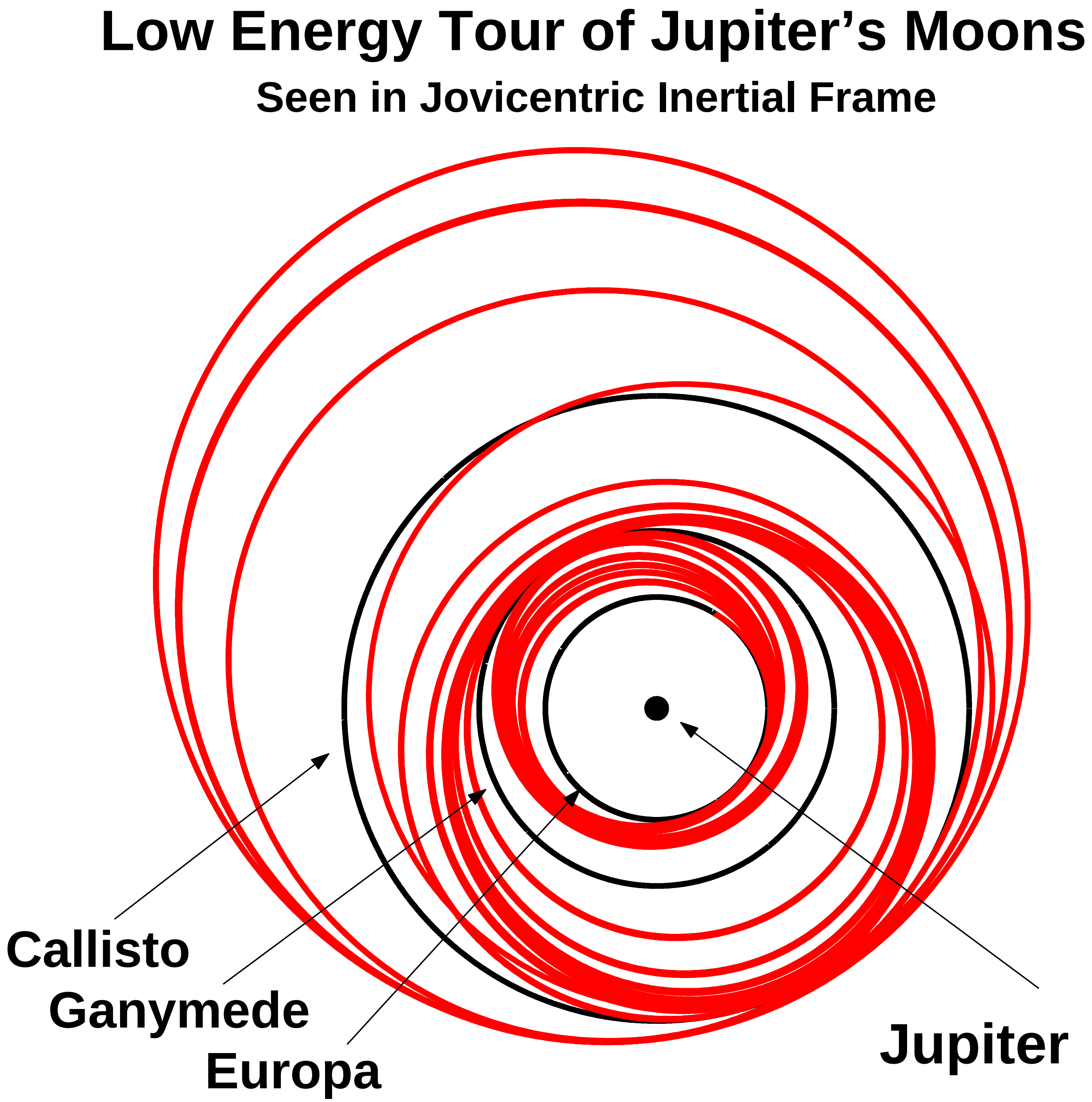
GEO to Moon Orbit Transfer

Seen in Geocentric Inertial Frame



TOF = 63 days
ΔV = 1211 m/s
1 Day Tick Marks

Low Energy Transfers



Common Link

- Consider a dust particle and a spacecraft.
- Gravity acts upon both primarily through the action of resonances and close encounters with other bodies $\Longrightarrow$ **complicated conservative dynamics**
- Add a significant perturbation
  - dust: dissipative radiation forces and radiation pressure
  - spacecraft: impulsive maneuvers or continuous low-thrust

  $\Longrightarrow$ **even more complicated!**
- **Good news:**

  Similar tools from nonlinear dynamics can be brought to bear on both.

Outline

- ***Dust Orbital Evolution***
  - ☐ Review problem
    - Gaps in the theory
  - ☐ Apply dynamical systems techniques
    - Break up N-body problem into 3-body subproblems
    - Phase space structures governing transport
    - Goal: statistical quantities (e.g., rates)

Outline

- ***Spacecraft Trajectory Design***
  - Apply same techniques
    - View as optimal control problem
    - Goal: minimize fuel consumption (ΔV)
    - Constraint: time of flight is reasonable

Dust Orbital Evolution

- ☐ Radiation forces affecting a small particle are parameterized by

$$\beta = \frac{\text{radiation pressure force}}{\text{stellar gravitation force}} \propto \frac{1}{D}$$

  - **Radiation pressure**

$$M_\star \rightarrow M_\star(1-\beta)$$

  - **Poynting-Robertson drag** (PR drag)

$$\dot{a}, \dot{e} \propto -\beta$$

where a = semimajor axis and e = eccentricity of particle

- ☐ No planets ⇒ orbital decay from 1 AU $\sim$ 10,000 years

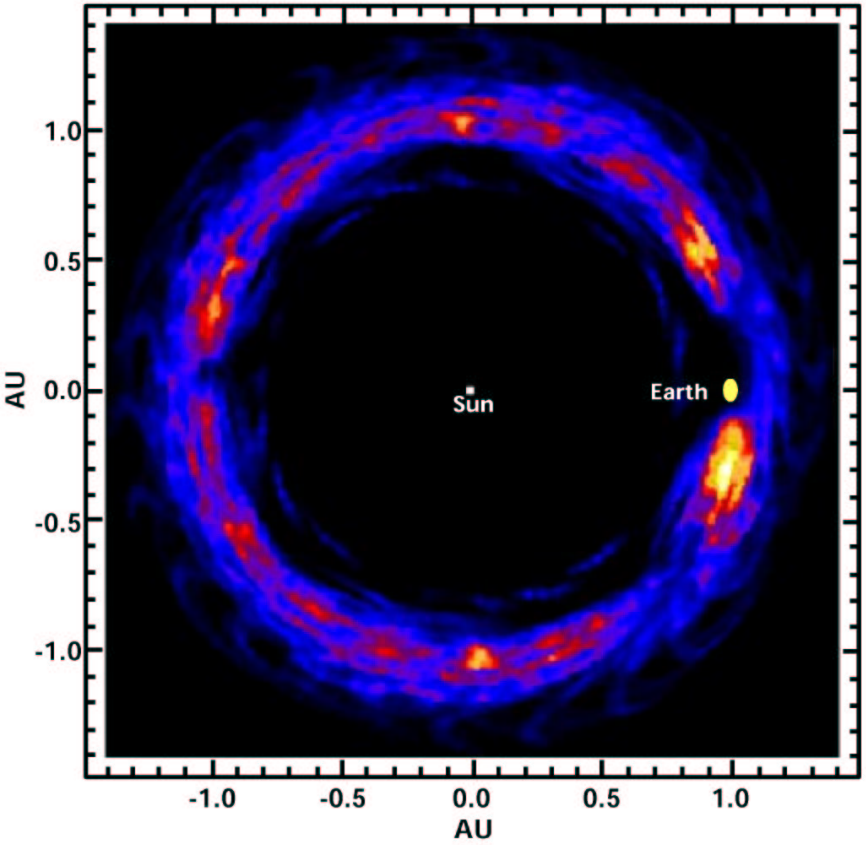
Dust Orbital Evolution

☐ Planets present ⇒ trapping into mean motion resonances (MMRs) and gravitational scattering via close encounters

- "Trapped": PR drag is counterbalanced by resonant gravitational perturbations
- Exterior MMRs most important
- Smaller $\beta \Rightarrow$ trapped in MMRs easier, stay trapped longer
- Resonance capture probability depends on e and argument of pericenter (Lazzaro, Sicardy, Roques, and Greenberg [1994])

Dust Orbital Evolution

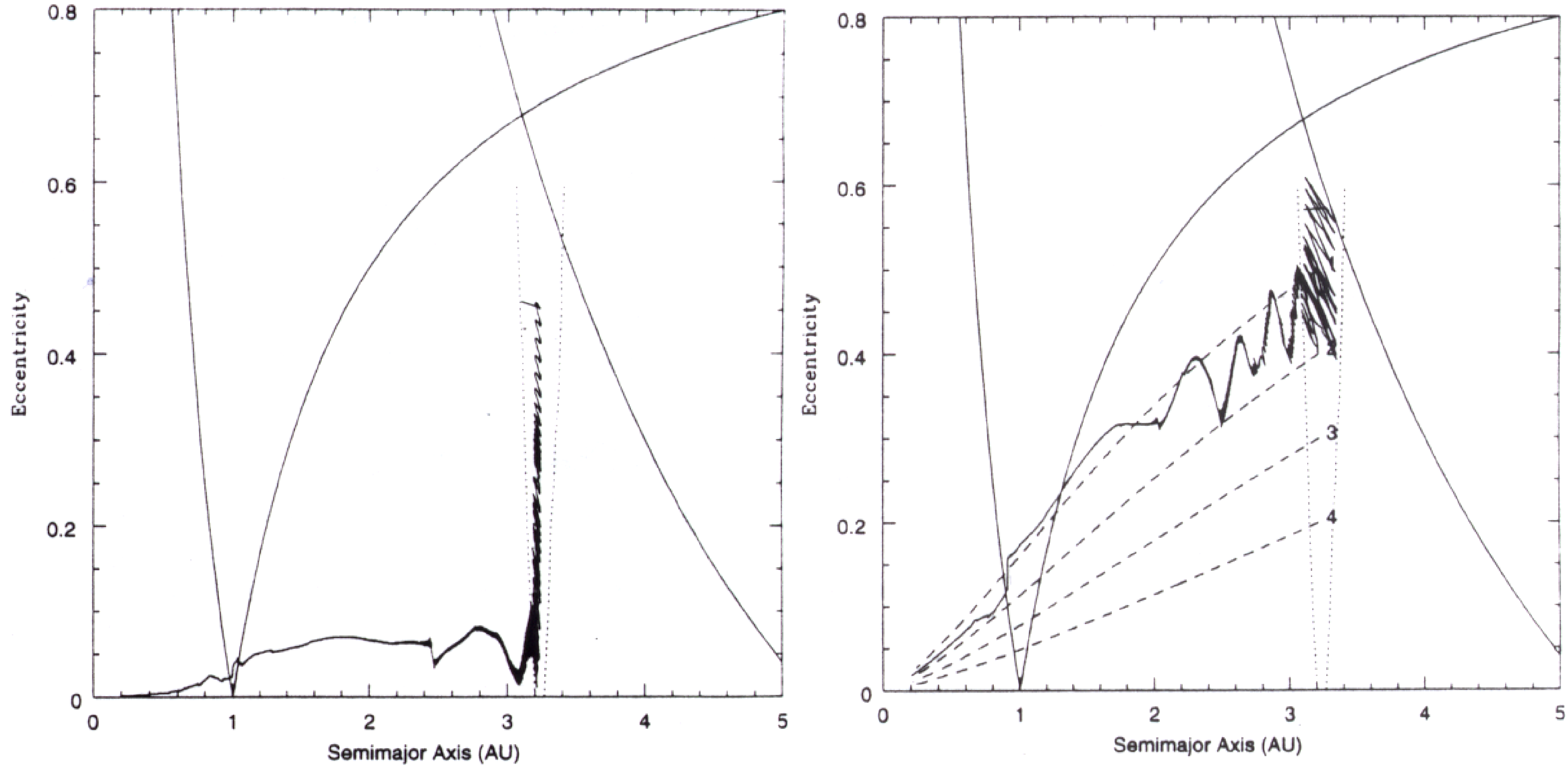
- **Numerical simulations** verify that dust grains get temporarily captured in MMRs creating a ring structure – the circumstellar disk.



Source: Dermott, Jayaraman, Xu, Gustafson, and Liou [1994]

Dust Orbital Evolution

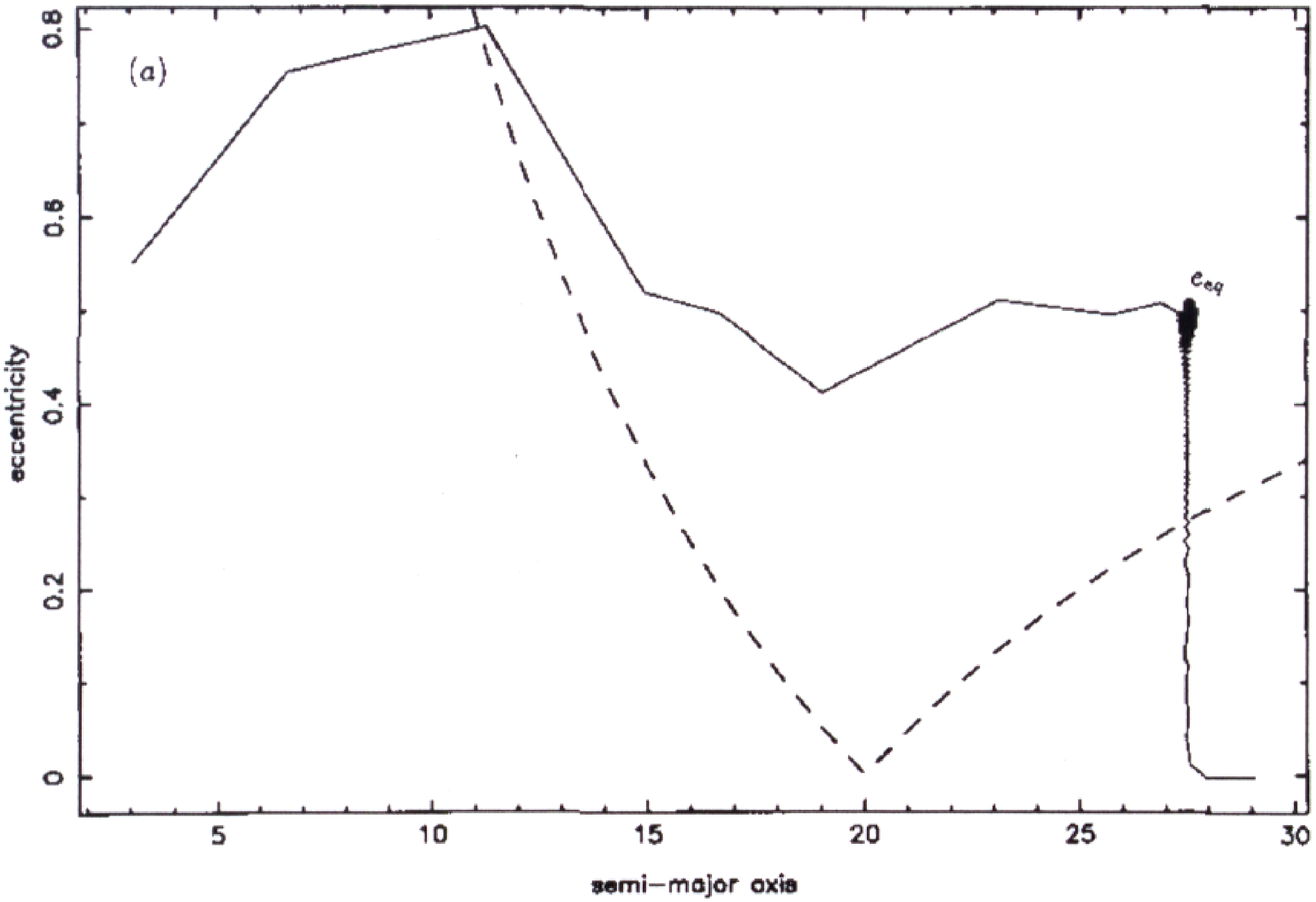
Particles are trapped in a MMR only temporarily.
Some may migrate starward toward another MMR.



Source: Liou and Zook [1996]

Dust Orbital Evolution

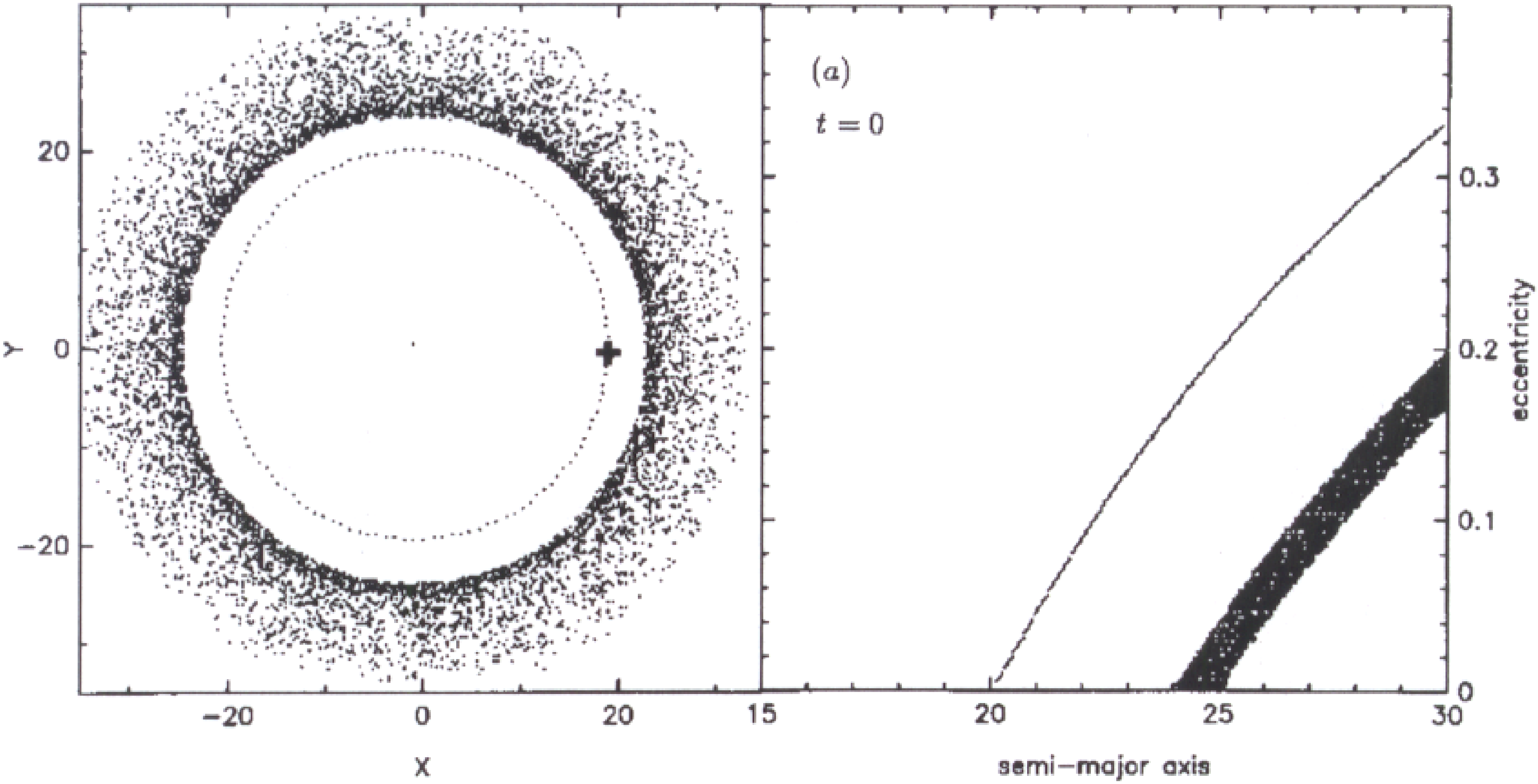
- Some increase in eccentricity and collide with the star.



Source: Roques, Scholl, Sicardy, and Smith [1994]

Dust Orbital Evolution

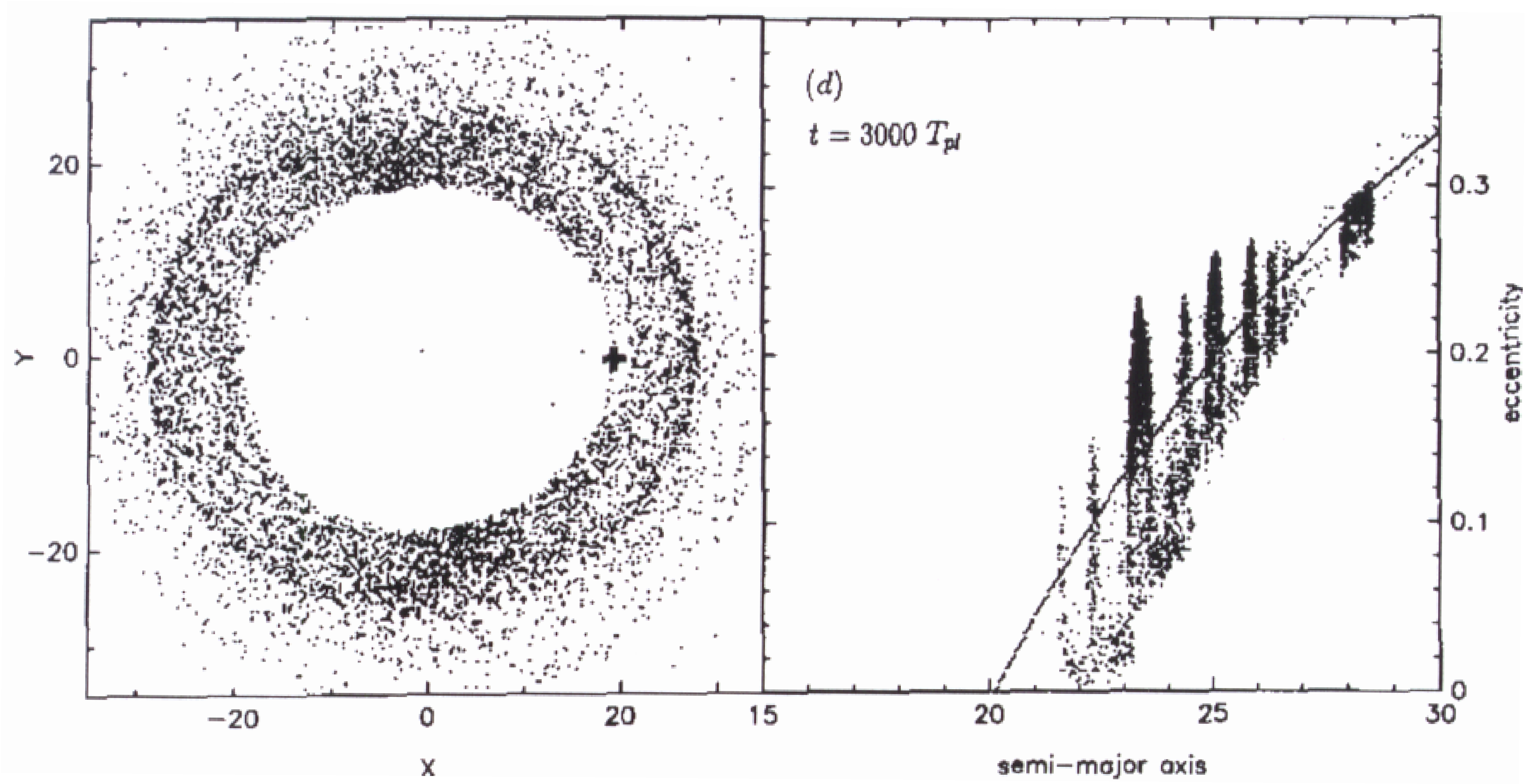
☐ Consider the evolution of a ring around β Pictoris.



Source: Roques, Scholl, Sicardy, and Smith [1994]

Dust Orbital Evolution

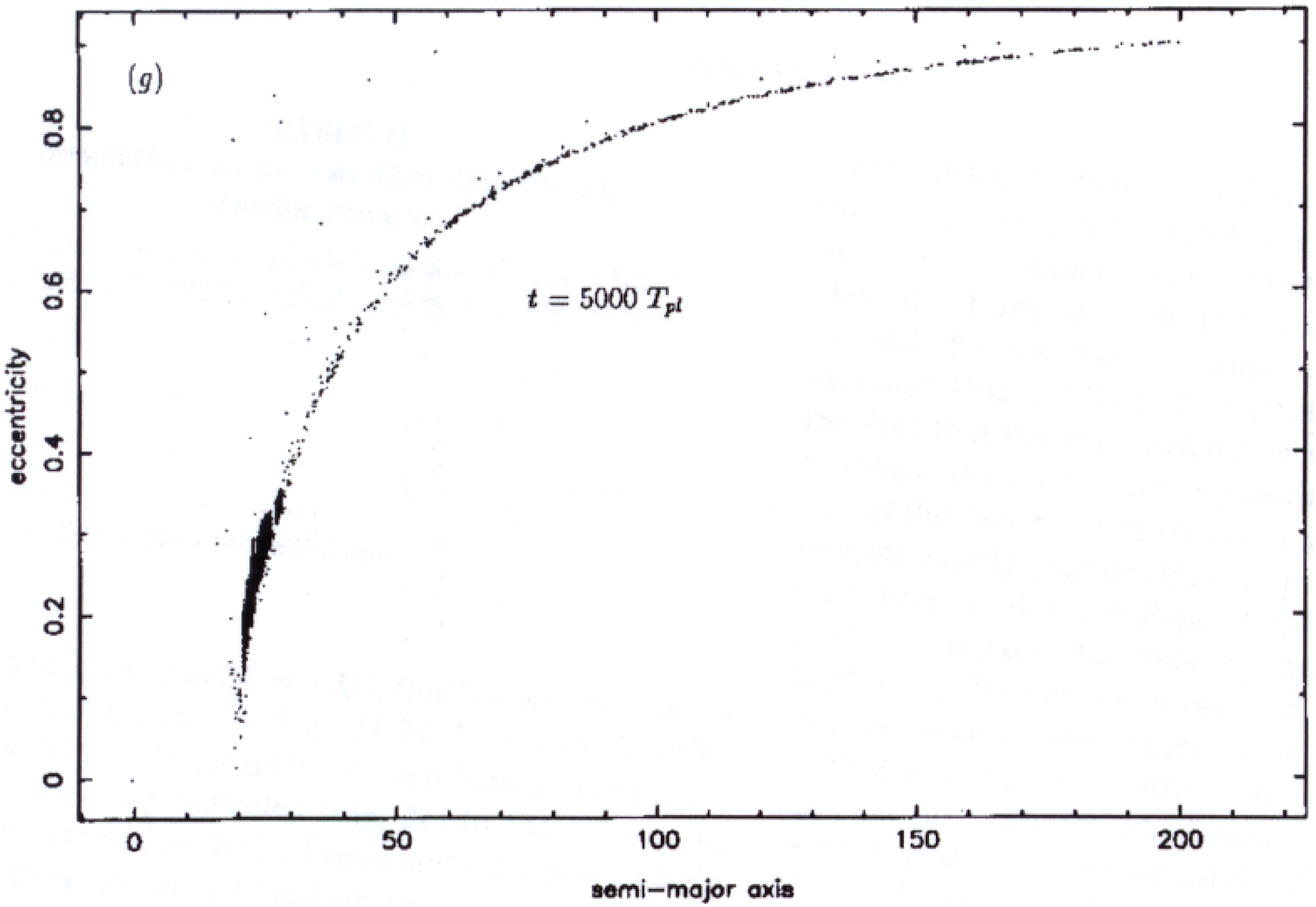
- ☐ Many particles become trapped in MMRs.



Source: Roques, Scholl, Sicardy, and Smith [1994]

Dust Orbital Evolution

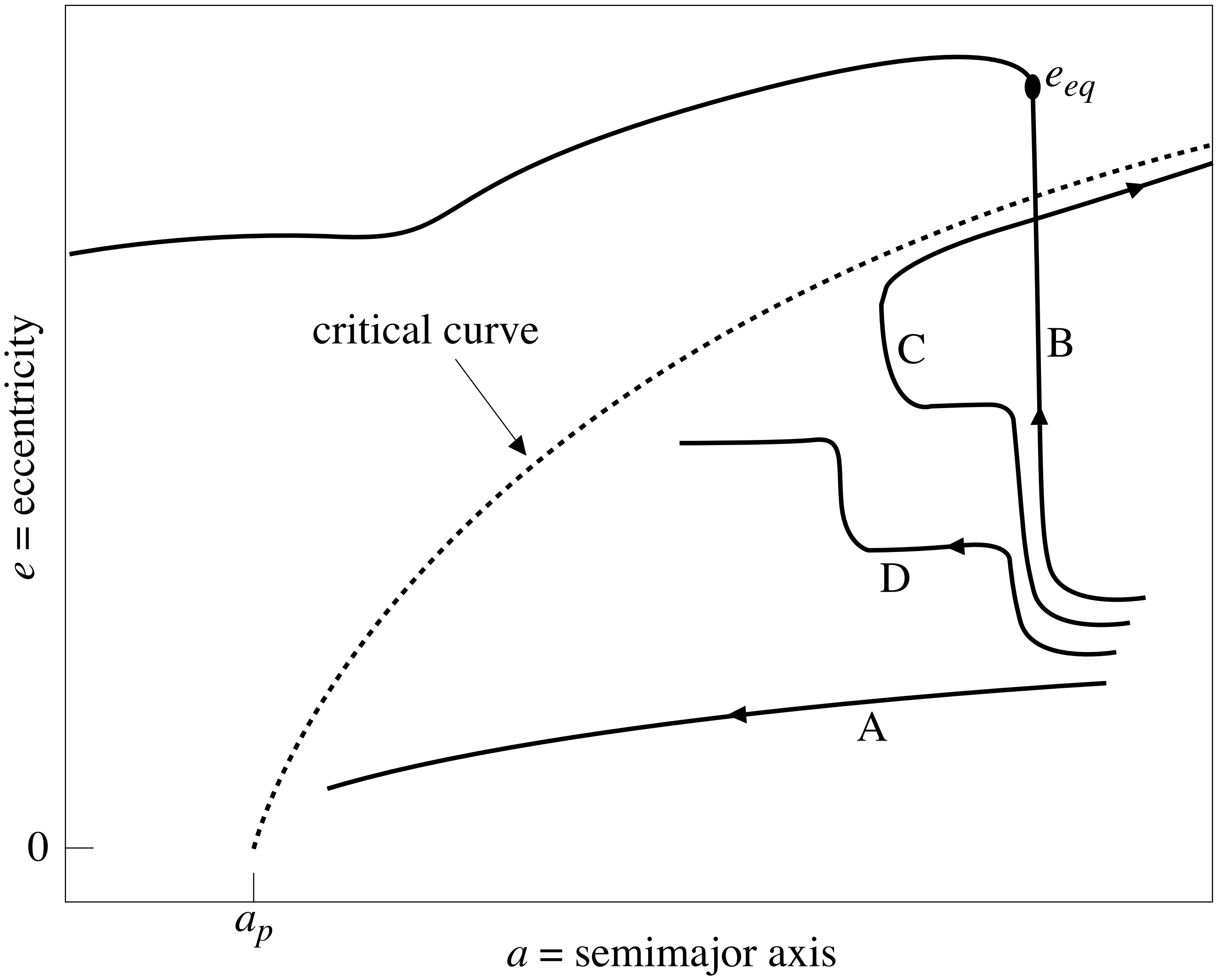
- ☐ Others are scattered by the planet to great distances.



Source: Roques, Scholl, Sicardy, and Smith [1994]

Gaps in the Theory

☐ A variety of behaviors are not well understood.

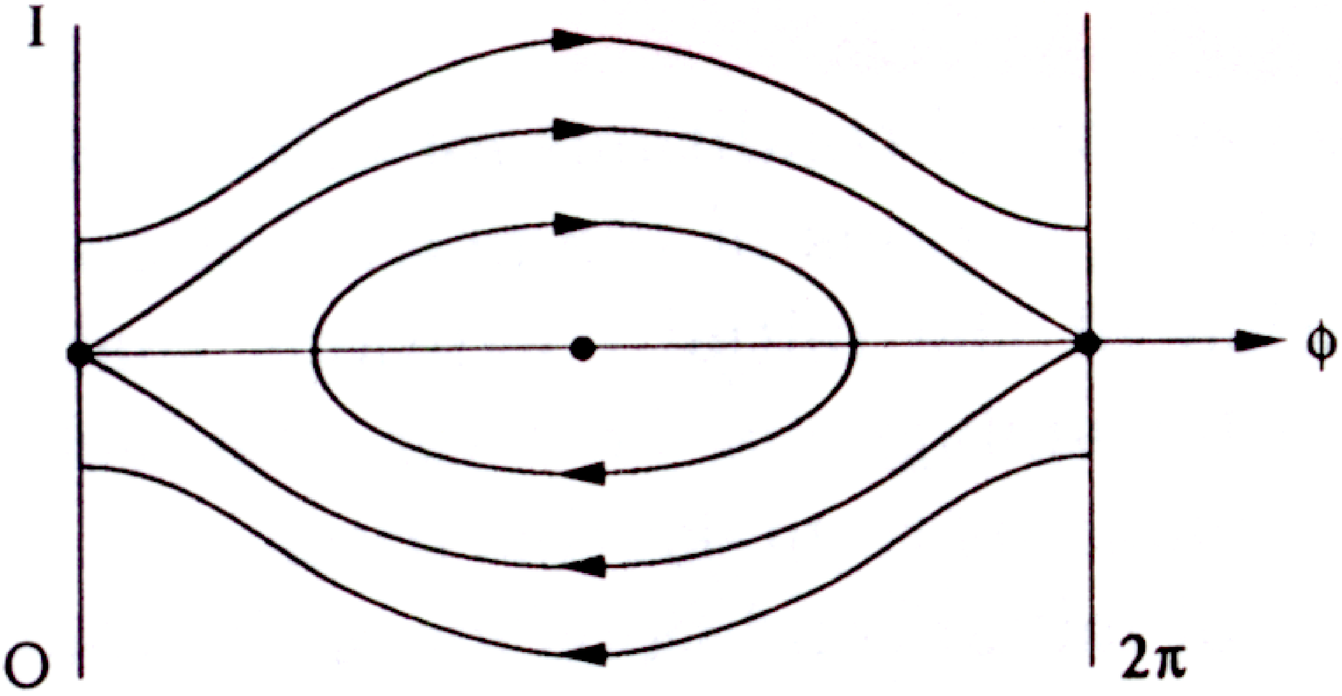


Gaps in the Theory

- Dissipative effects combined with resonance phenomena are known to lead to complex dynamics (Lazzaro, Sicardy, Roques, and Greenberg [1994]).
- Much progress has occurred in recent years, but there are still gaps in the theory which need addressing.
- In particular, the related phenomena of **jumping between resonances** with a planet during migration toward a star and the outcomes of **close encounters** with planets have not been considered in any theory of dust orbital evolution. (Dermott, Grogan, Durda, Jayaraman, Kehoe, Kortenkamp, and Wyatt [2001]).

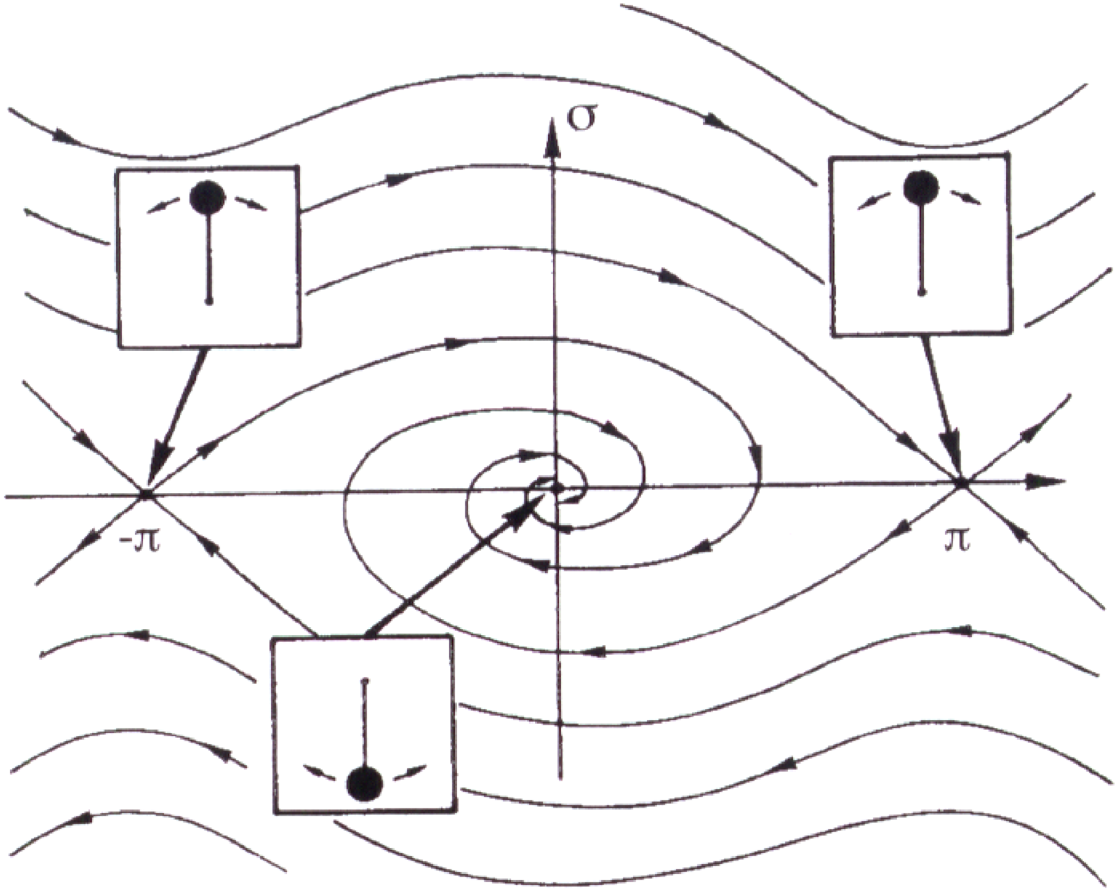
Transport near a MMR

- ☐ Analytical studies of capture into resonance have been performed (e.g., Beaugé and Ferraz-Mello [1994]). Evolution near a resonance is modeled by a pendulum-like Hamiltonian with slowly varying parameters.



Transport near a MMR

- As slowly varying parameters change, the homoclinic orbits generically break up, and particles may get captured into the resonance region or pass out of it.



Transport near a MMR

- Questions motivating such study are:
    - Is capture into resonance possible?
    - What is the probability of capture into resonance?
    - What is the average time spent within a resonance?
- Much progress has been made in this area (e.g., Wisdom [1982,1983], Borderies and Goldreich [1984]).
- But study has focused on the **local** dynamics around a single resonance.

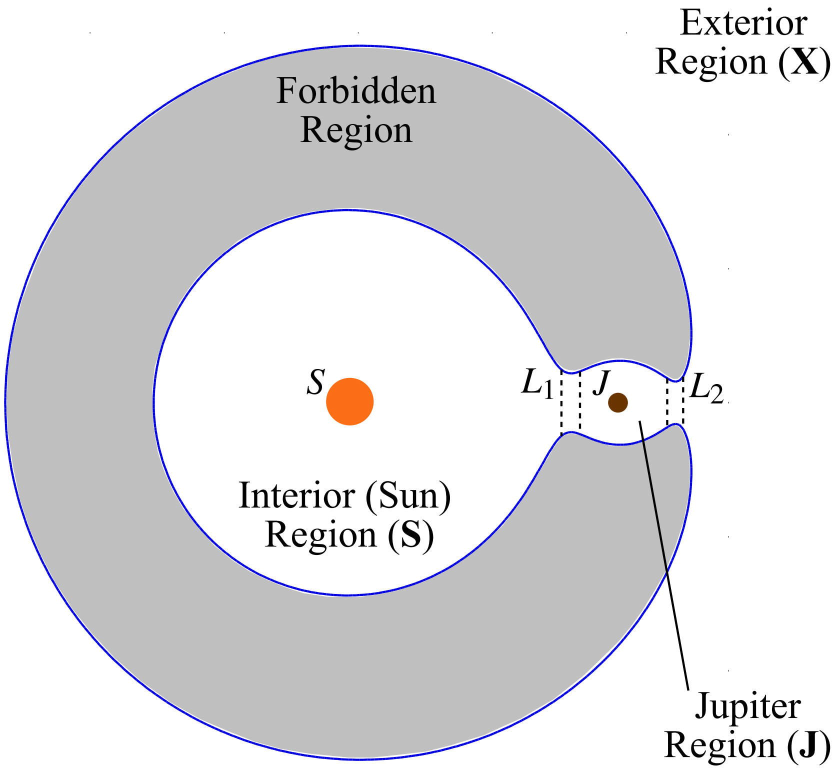
Transport between MMRs

- Instead of looking at each MMR in isolation, our view is to consider the entire **global** phase space picture of all MMRs.
  - Only in the global setting can one compute the transport rates between different MMRs.
- First step: consider the conservative (Hamiltonian) **planar circular restricted three-body problem** (PCRTBP)

Transport between MMRs

Recall PCRTBP: motion of a particle in the gravitational field of two larger bodies in circular motion.

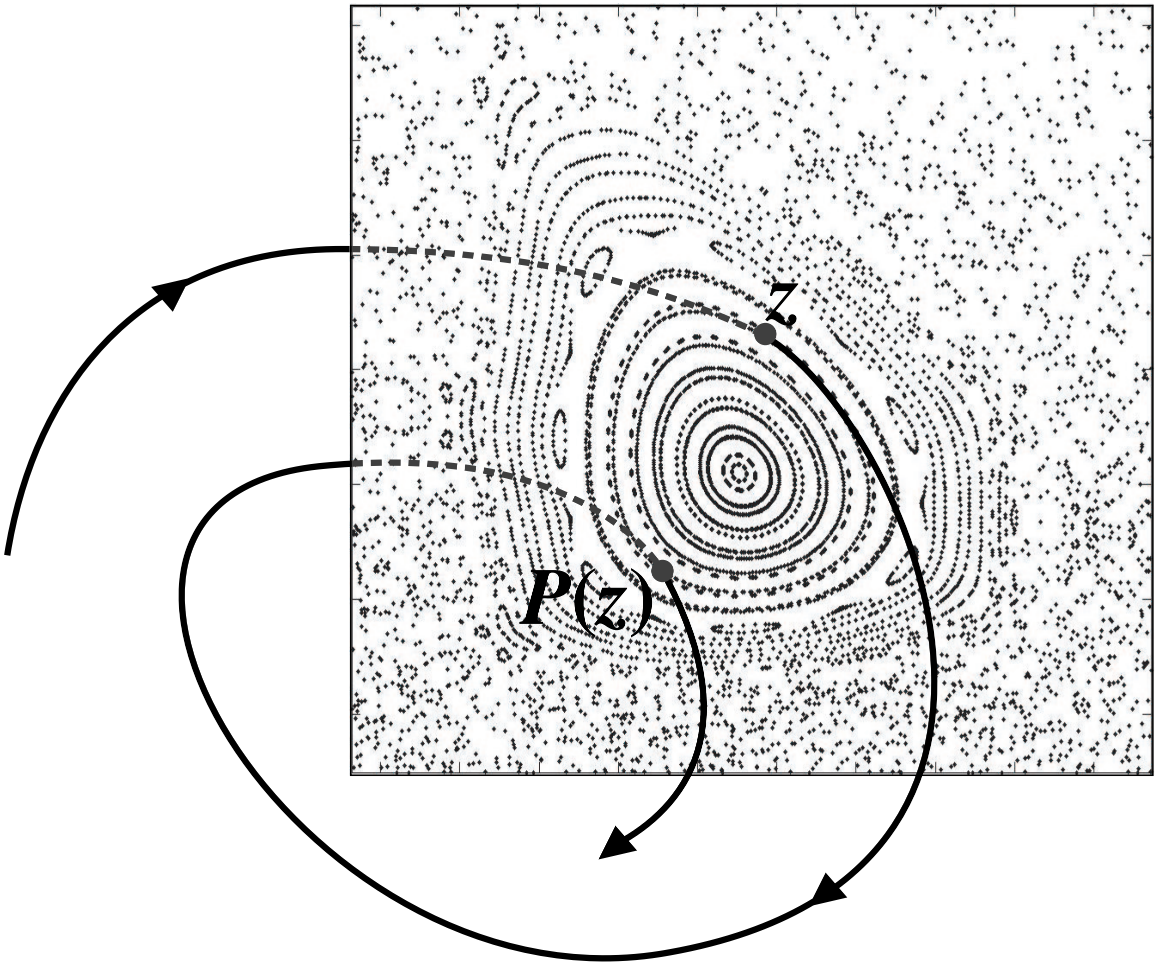
- View in rotating frame $\Longrightarrow$ time-independent $\Longrightarrow$ constant energy E



Rotating frame: different regions of motion at energy E.

Transport between MMRs

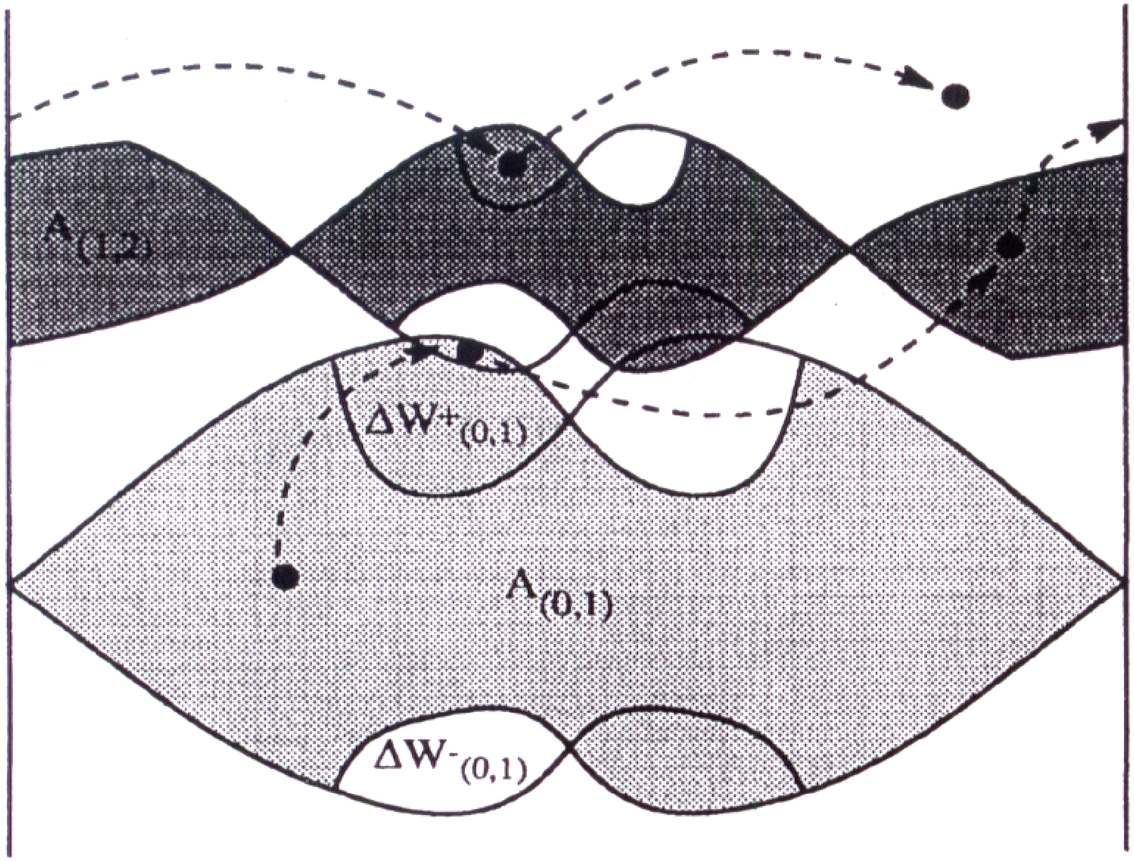
Study Poincaré surface of section at fixed energy E, reducing system to a 2-dimensional area preserving map.



Poincaré surface of section

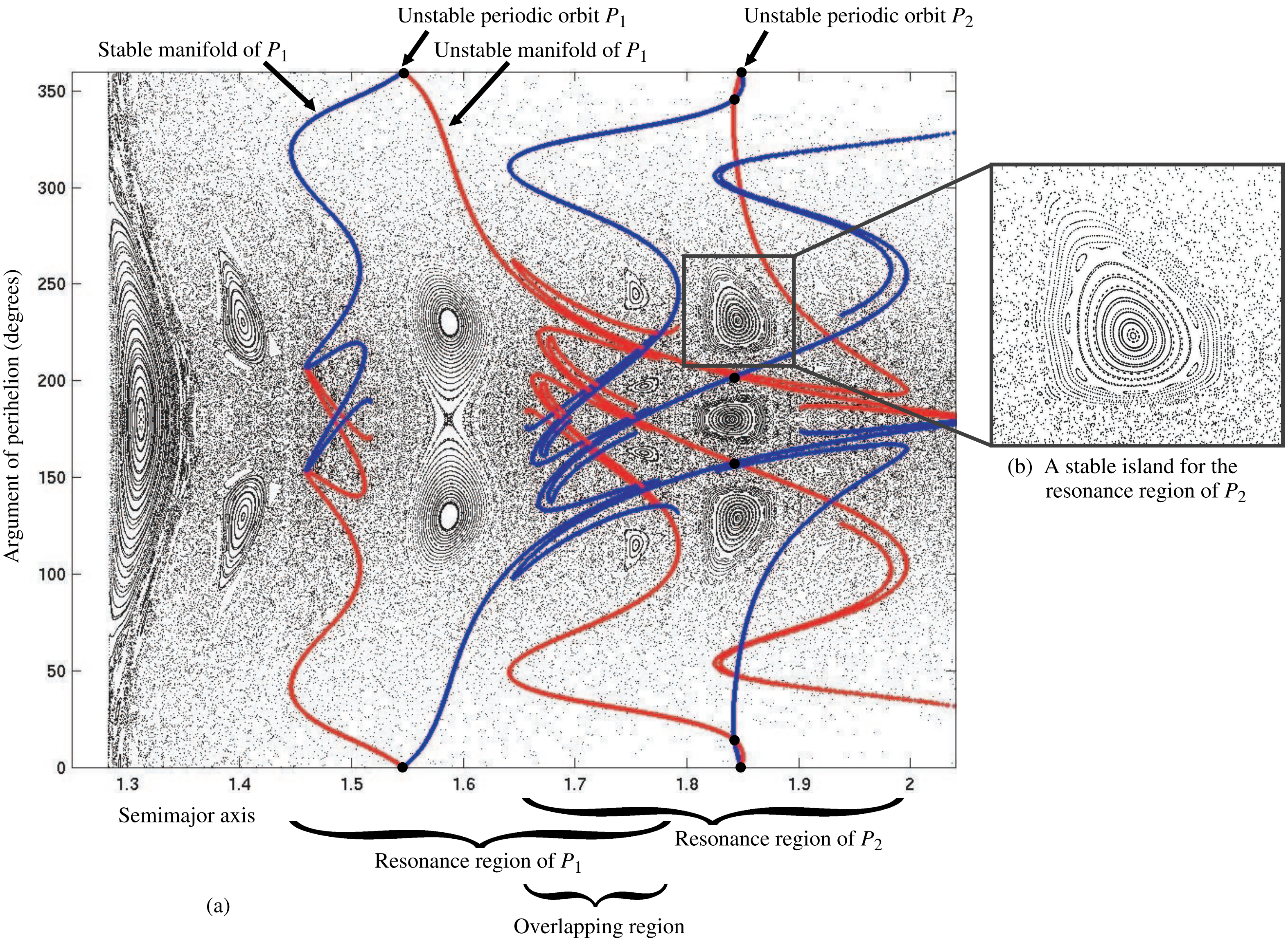
Transport between MMRs

In such a system, the natural transport is well understood as the movement of trajectories among resonances (see Meiss [1992], Schroer and Ott [1997]).



Transport between MMRs

We can compute the resonance regions for the PCRTBP.



(b) A stable island for the resonance region of P_2

Transport between MMRs

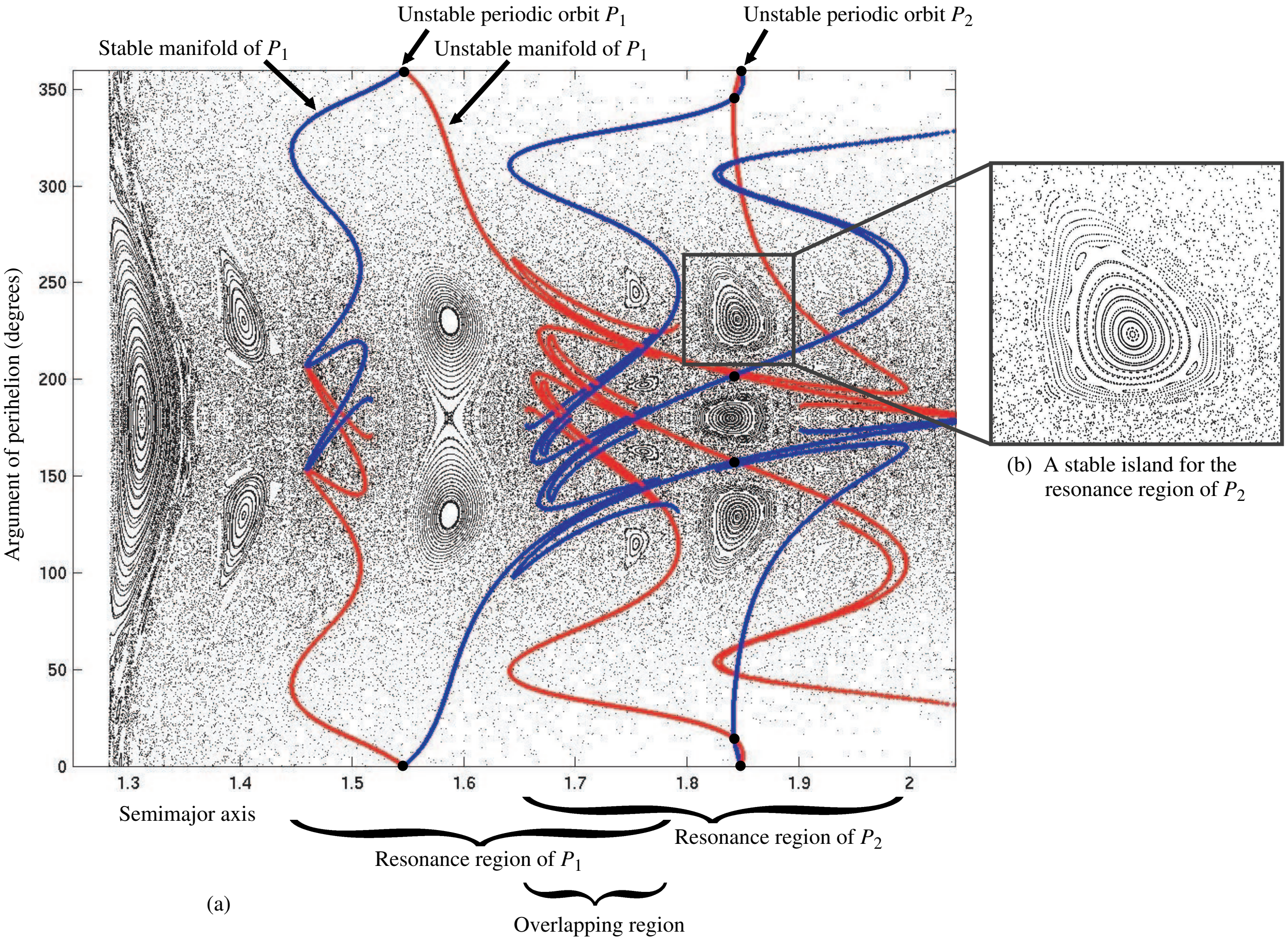
- **The transport problem:**

  Suppose the $p:q$ MMR has an initial population of $N_{(p:q)}$ points. The goal of our transport description is to determine the population of each MMR after t iterations
  (see MacKay, Meiss, and Percival [1984]).

- In order to leave the $p:q$ MMR, a point must fall in the exit lobe of either the left or right turnstile. There is a turnstile in only one island of the chain of $|p-q|$ islands.

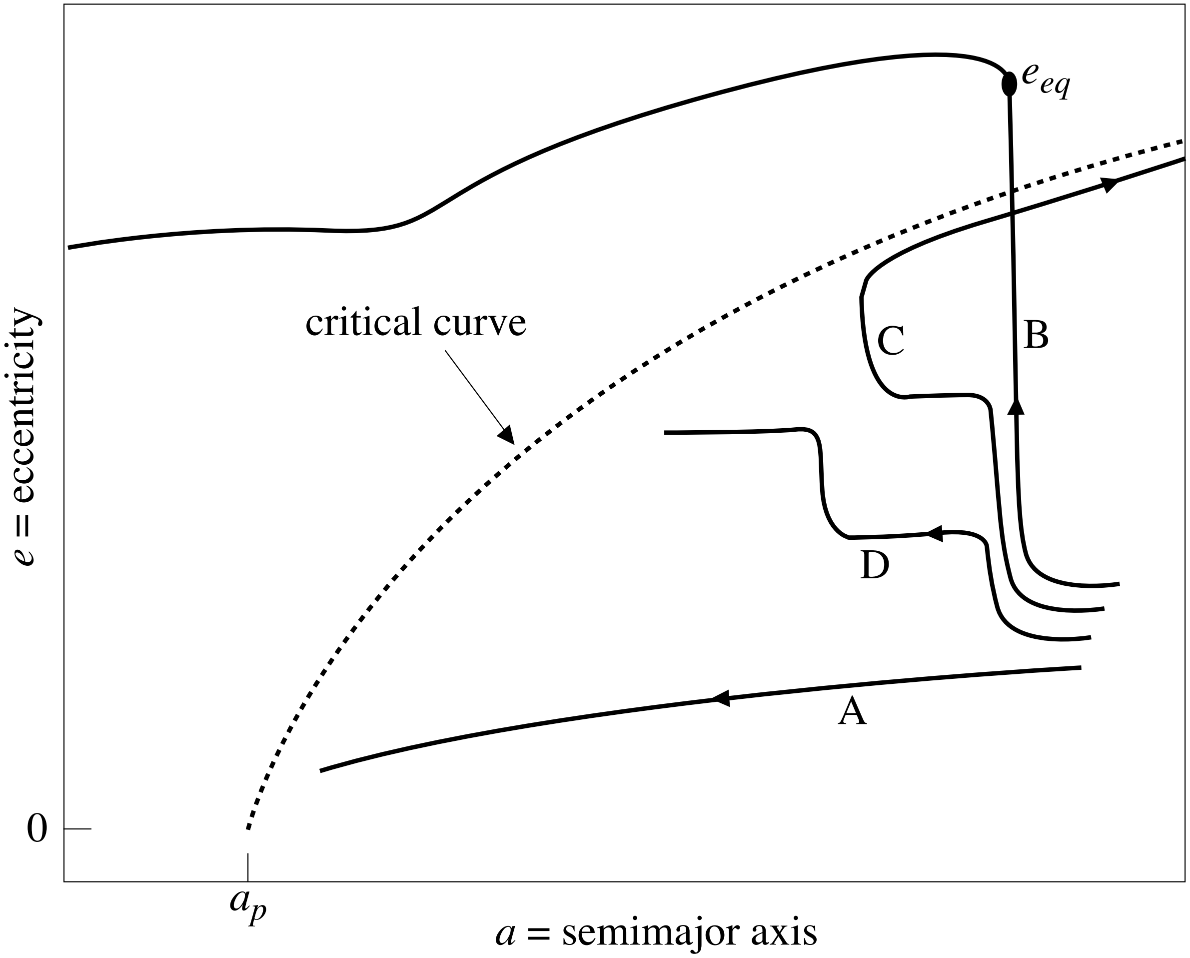
Transport between MMRs

A direct transition from a $p : q$ to a $p\prime : q\prime$ MMR is possible only if the exit lobe of a $p : q$ turnstile overlaps with the entry lobe of a $p\prime : q\prime$ turnstile.



Close Encounters

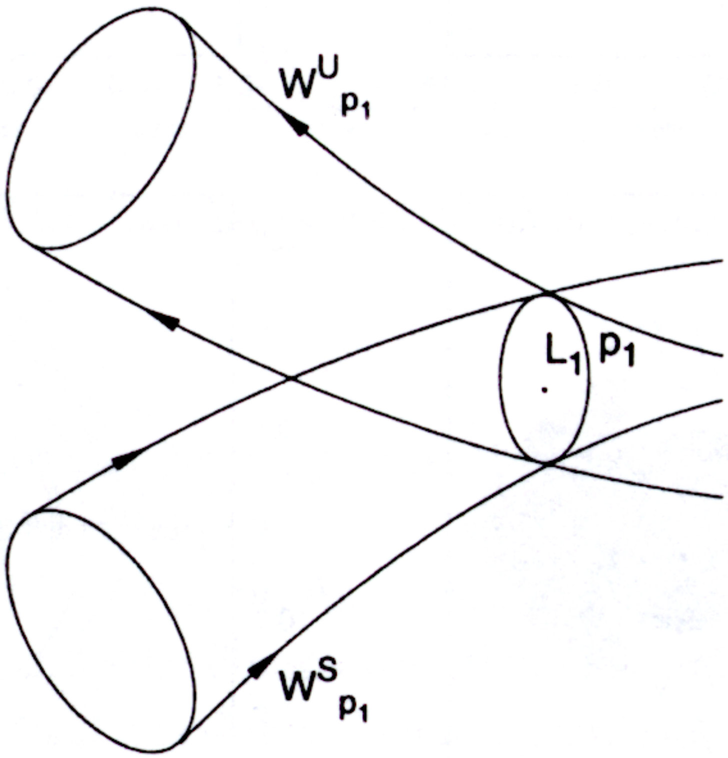
For a particle near the planet-crossing critical curve, the possibility for a **close encounter** with the planet becomes possible.



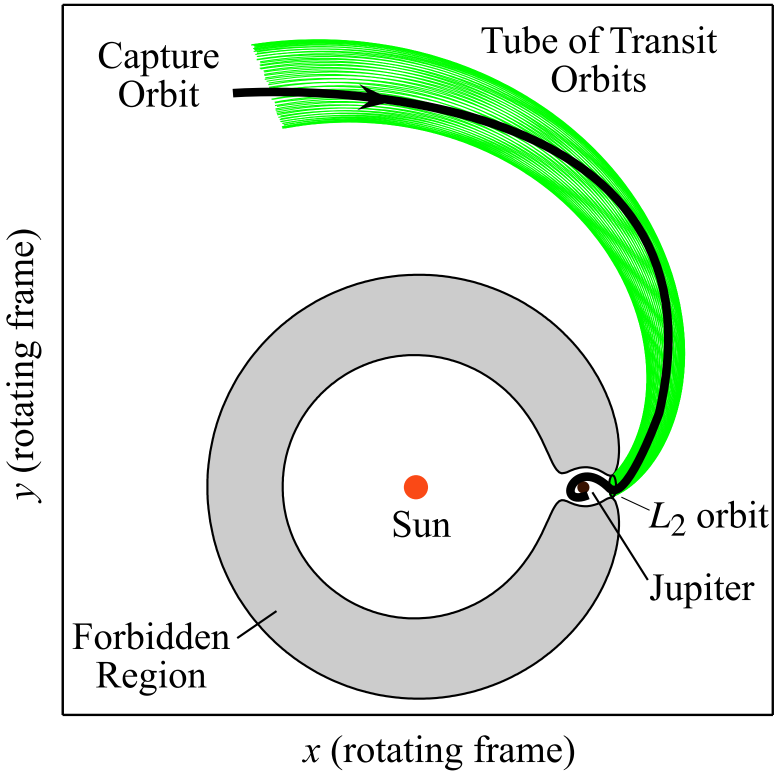
Close Encounters

This is mediated by **tubes** of transit orbits, heading toward (or away from) the planetary region.

- the stable and unstable manifolds of periodic orbits about L_1 and L_2 (see Koon, Lo, Marsden, SDR [2000])



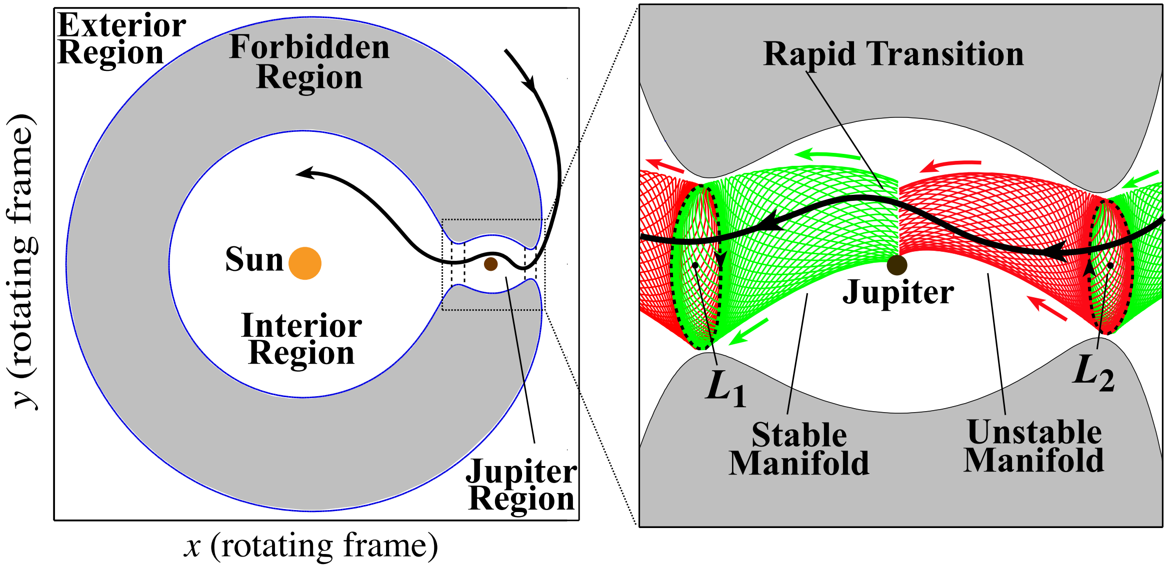
In phase space (schematic)



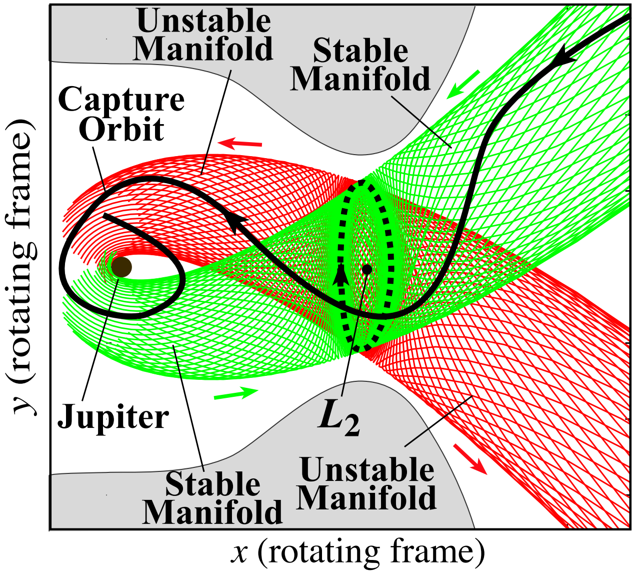
In position space

Close Encounters

A particle may pass by the planet or be temporarily captured in orbit about the planet.



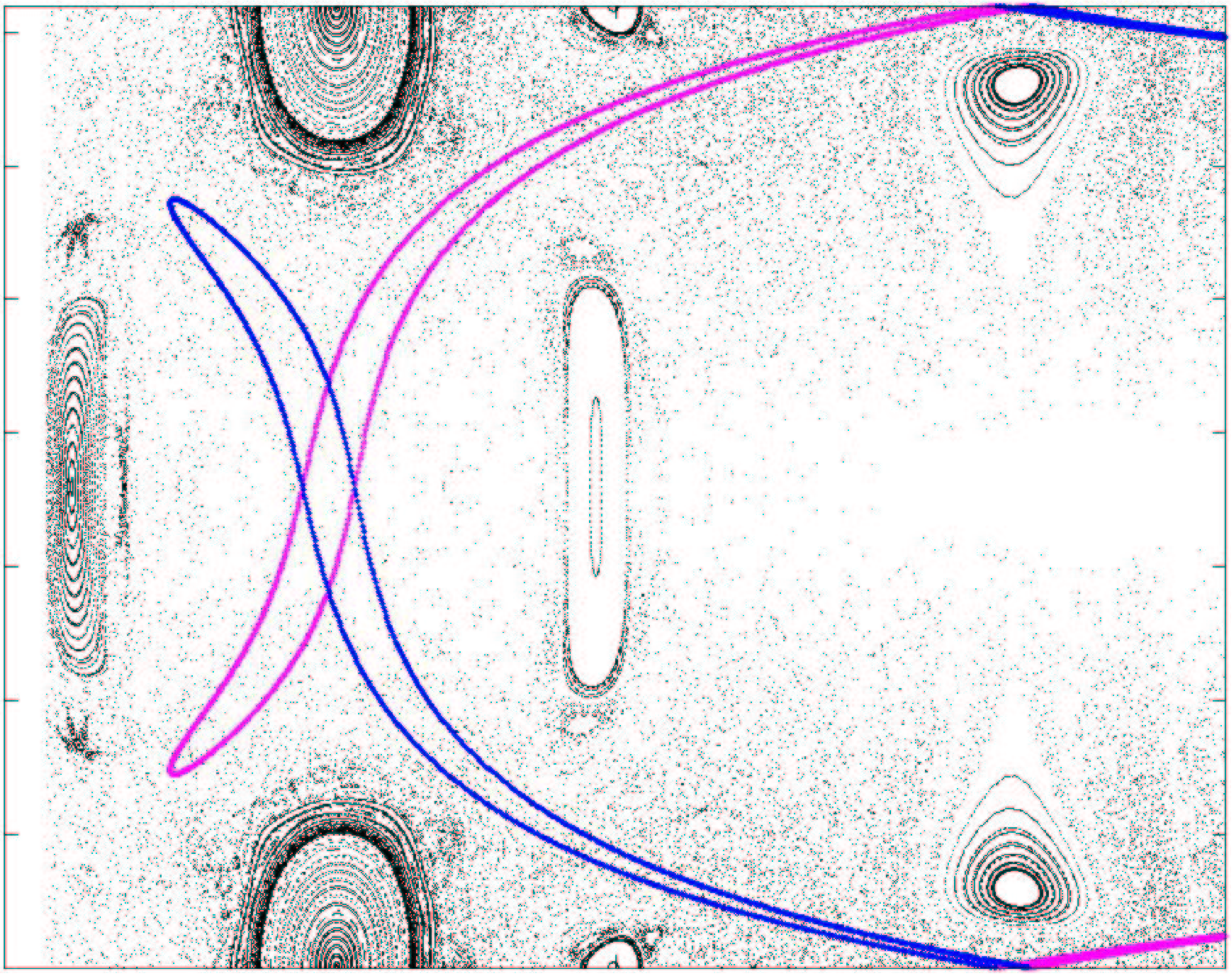
(a)



(b)

Close Encounters

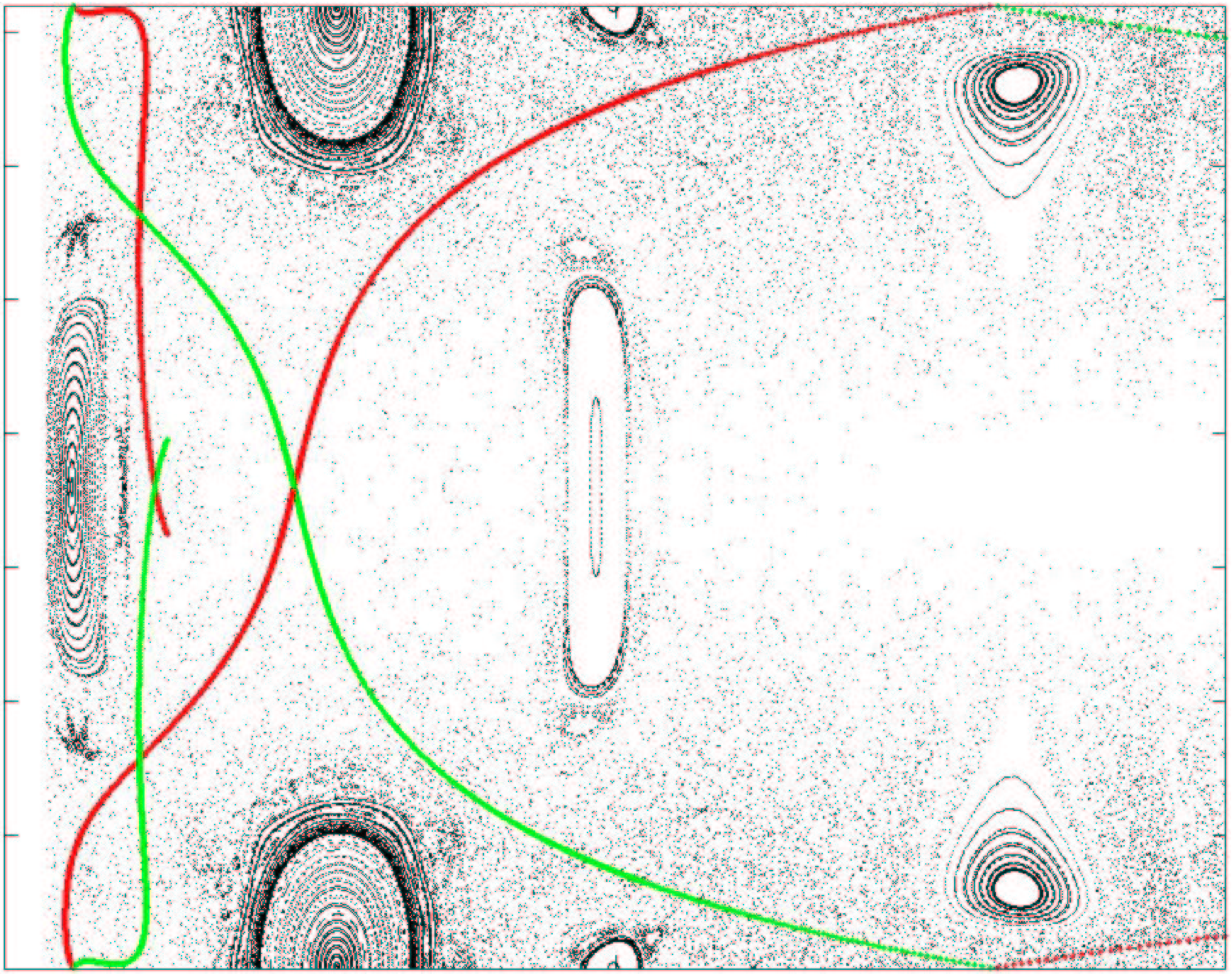
- Poincaré section: tube cross-sections are closed curves.



Particles inside curves move toward or away from Jupiter

Close Encounters

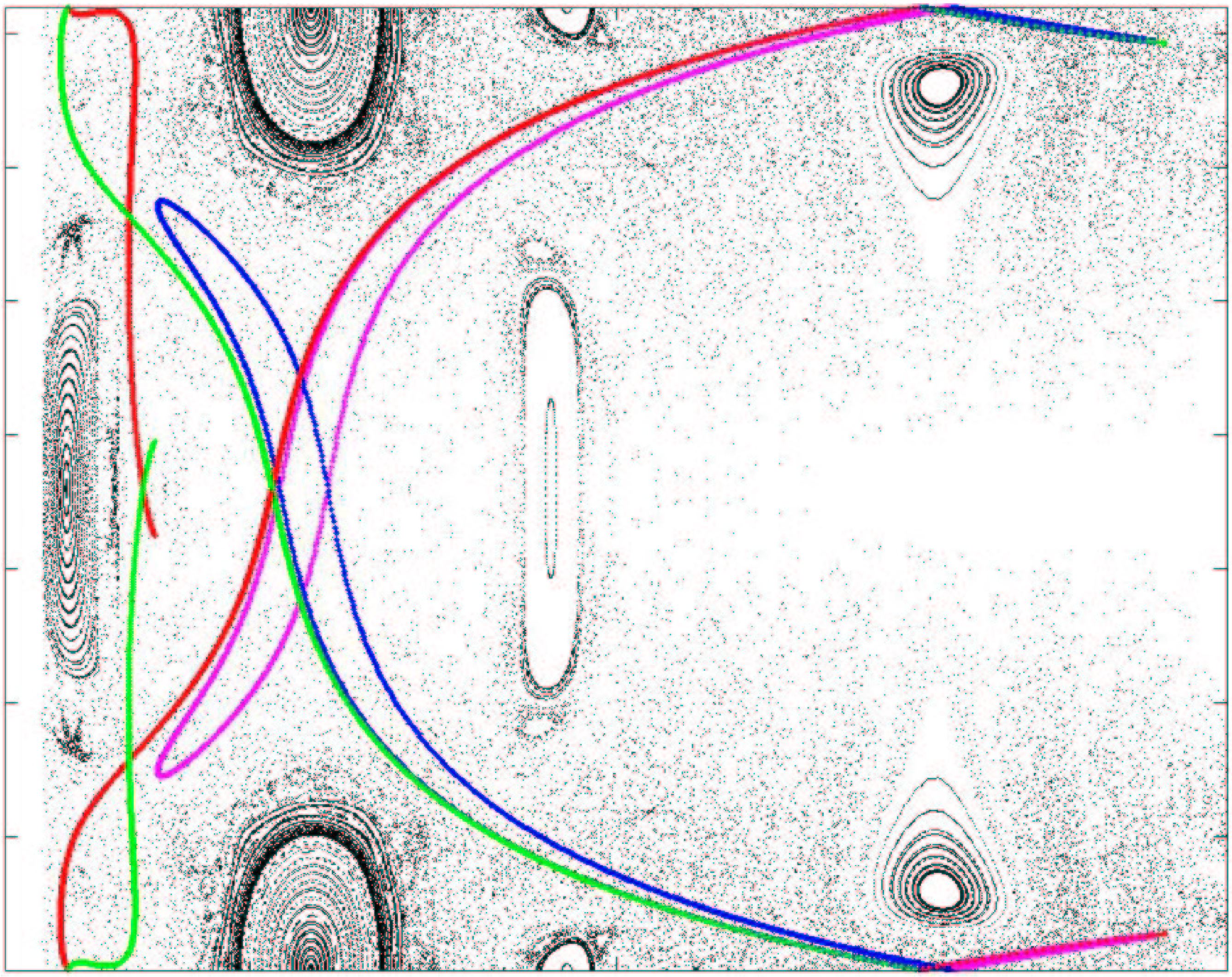
- ☐ Same Poincaré section: plot resonance regions.



2:3 exterior MMR with Jupiter

Close Encounters

- Regions of overlap lead to close encounters.



Regions of overlap occur

Statistical Quantities

- Using this **lobe dynamics** approach (see Wiggins [1992]), several statistical quantities of interest can be computed as a function of planetary mass and particle energy.
  - average trapping time in a $p:q$ MMR
  - flux entering $p:q$ MMR from $p\prime:q\prime$ MMR

Drag Perturbed Case

- This approach must be augmented to consider PR drag ($\beta > 0$).
  - Little theory is known regarding the effect of drag on Hamiltonian systems.
  - Kirk, Marsden, and Silber [1996] suggest the use of Hamiltonian methods even in the presence of drag is promising.
  - Numerical evidence suggests some phase space structure governing transport of dust between MMRs persists even for large β (Roques, Scholl, Sicardy, and Smith [1994]).

Drag Perturbed Case

☐ Particles migrate to different energies.

- $\dot{E} < 0$ in interior region $\Rightarrow$ collide with star
  $\dot{E}$ can be $\pm$ in exterior region
  Liou, Zook, and Jackson [1995]

☐ Remnants of conservative phase space structure likely survive.

- e.g., boundaries defining resonance regions, turnstiles

☐ For small $\beta > 0$, symmetry will be broken

- e.g., motion tends starward

$\Longrightarrow$ More numerical experiments and theory needed

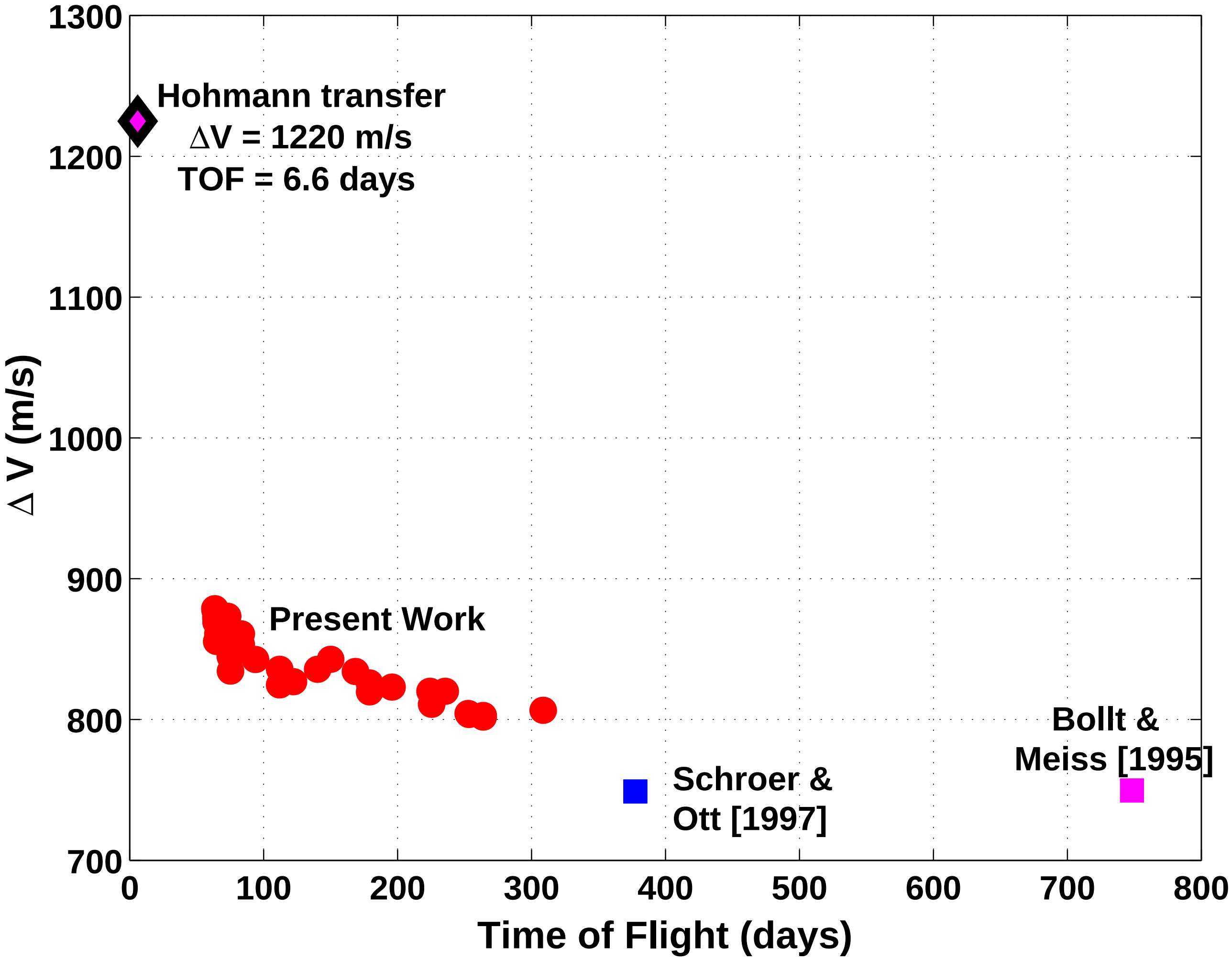
Trajectory Design

- ☐ Using the same dynamics, spacecraft trajectories can be designed
  - Use natural dynamics to lessen propellant consumption
- ☐ Consider a transfer from Earth orbit to lunar orbit
  - Use PCRTBP as model
  - Bollt and Meiss [1995]: targeting through recurrence
  - Schroer and Ott [1997]: targeting passes between MMRs
- ☐ Current work: seek intersections between MMRs and tubes leading to ballistic capture by the moon
  - Take full advantage of all known phase space structures

Trajectory Design

☐ **Results**: much shorter transfer times than previous authors for only slightly more ΔV

Trajectories from a circular Earth orbit (r=59669 km) to a stable lunar orbit

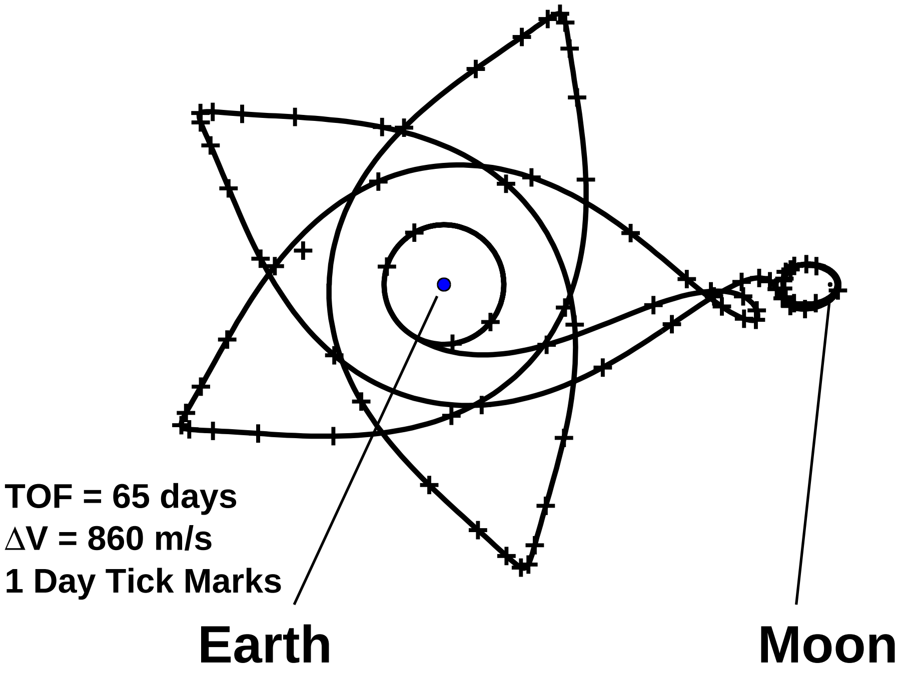


Trajectory Design

- Compare with Bollt and Meiss [1995]
  - A tenth of the time for only 100 m/s more

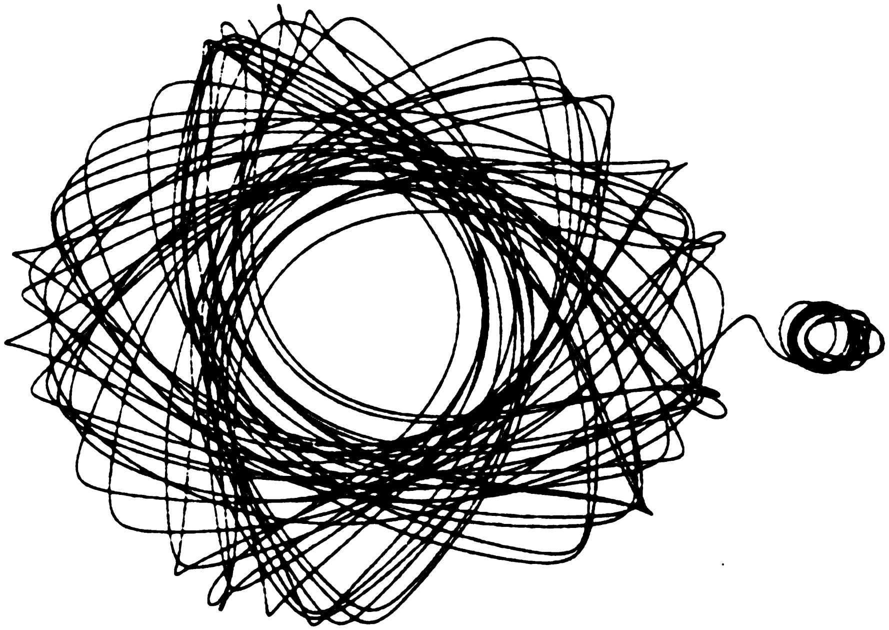
Current Result

65 days, $\Delta V = 860$ m/s



Bollt and Meiss [1995]

748 days, $\Delta V = 750$ m/s



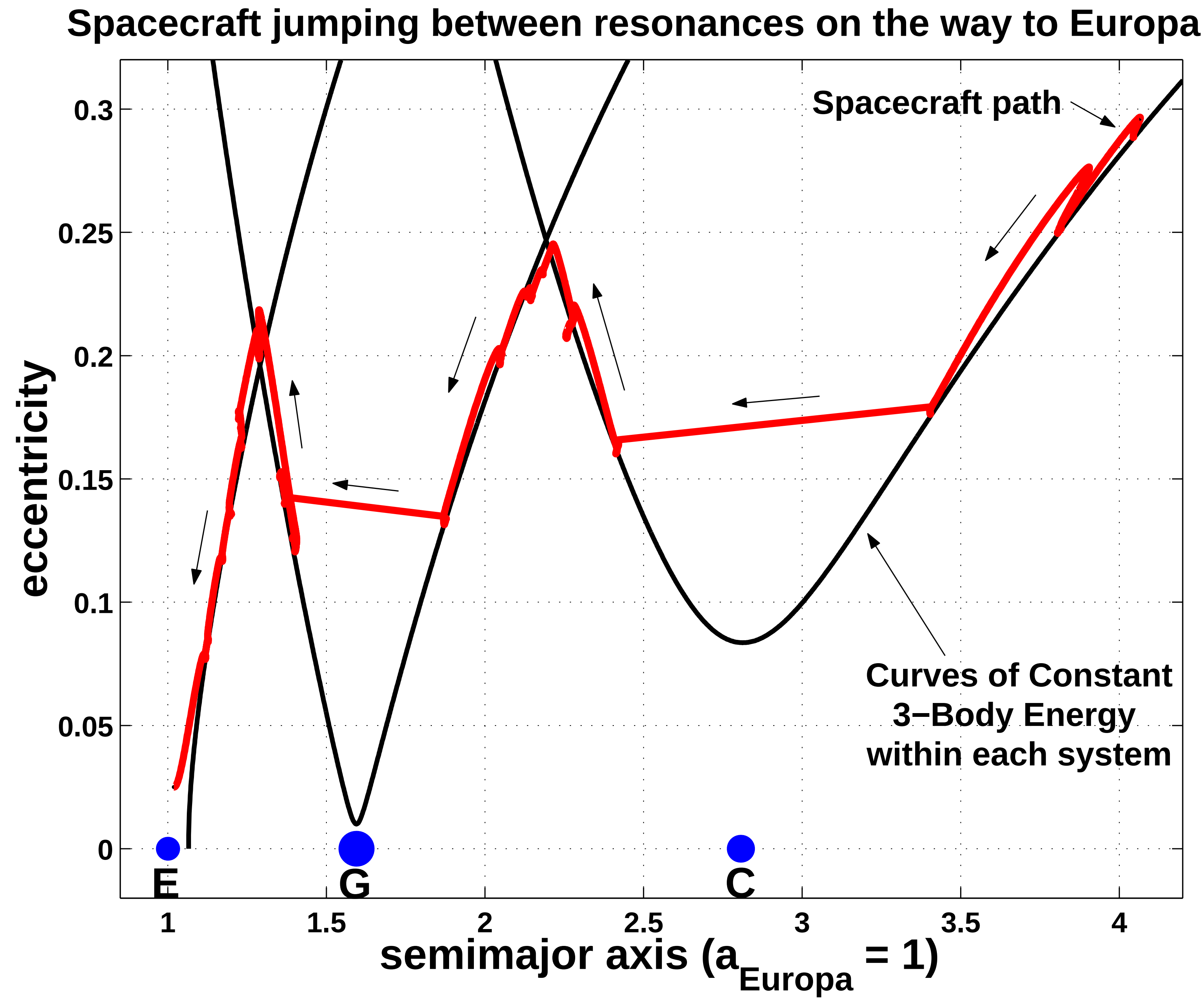
Trajectory Design

- One can consider jumping between resonances of two 3-body systems.
  - Decompose the N-body problem into successive coupled 3-body problems (Gomez, Koon, Lo, Marsden, Masdemont, SDR [2001]).

Trajectory Design

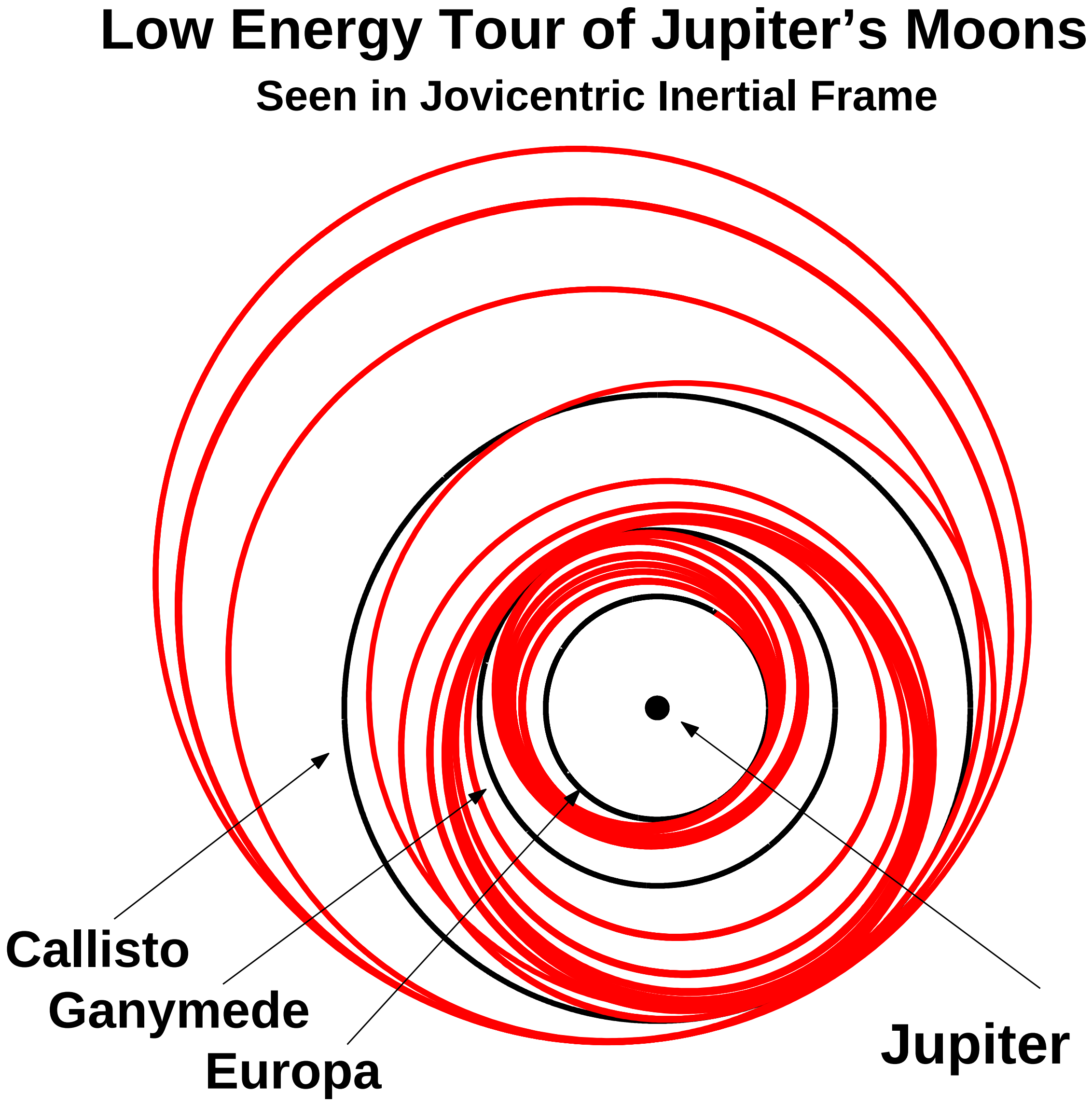
- Consider a trajectory to tour the moons of Jupiter
  - Begin in an eccentric orbit with perijove at Callisto's orbit
  - Suppose one wants to visit and orbit each of the moons
  - Using a standard patched-conics approach, the ΔV necessary may be prohibitively high
- Preliminary work suggests such a tour may be realizable for very little ΔV by jumping between MMRs of different moons and effecting ballistic captures

Trajectory Design



Trajectory Design

- ☐ For this tour: $\Delta V = 20$ m/s, but TOF is a few years



Trajectory Design

- As seen in the case of the Earth to lunar orbit transfer, time of flight can decrease dramatically with slightly increased ΔV
- More work needs to be done to determine the time-of-flight vs. ΔV curve using this approach.

References

Main Papers:

- Gómez, G., W.S. Koon, M.W. Lo, J.E. Marsden, J. Masdemont and S.D. Ross [2001] ***Invariant manifolds, the spatial three-body problem and space mission design.*** *AAS/AIAA Astrodynamics Specialist Conference.*
- Koon, W.S., M.W. Lo, J.E. Marsden and S.D. Ross [2001] ***Resonance and capture of Jupiter comets.*** *Celestial Mechanics and Dynamical Astronomy*, 81(1-2), 27–38..
- Koon, W.S., M.W. Lo, J.E. Marsden and S.D. Ross [2001] ***Low energy transfer to the Moon.*** *Celestial Mechanics and Dynamical Astronomy.* 81(1-2), 63–73.
- Koon, W.S., M.W. Lo, J.E. Marsden and S.D. Ross [2000] ***Heteroclinic connections between periodic orbits and resonance transitions in celestial mechanics.*** *Chaos* 10(2), 427–469.
- ***Targeting low energy trajectories to the Moon***, in preparation.

The End