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Chaotic Motion in the Solar System: Mapping the Interplanetary Transport Network

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California State University, Long Beach, February 24, 2003

Motivation

- Apply dynamical systems theory to determine the transport of minor bodies throughout the solar system.

Insert movie of asteriods

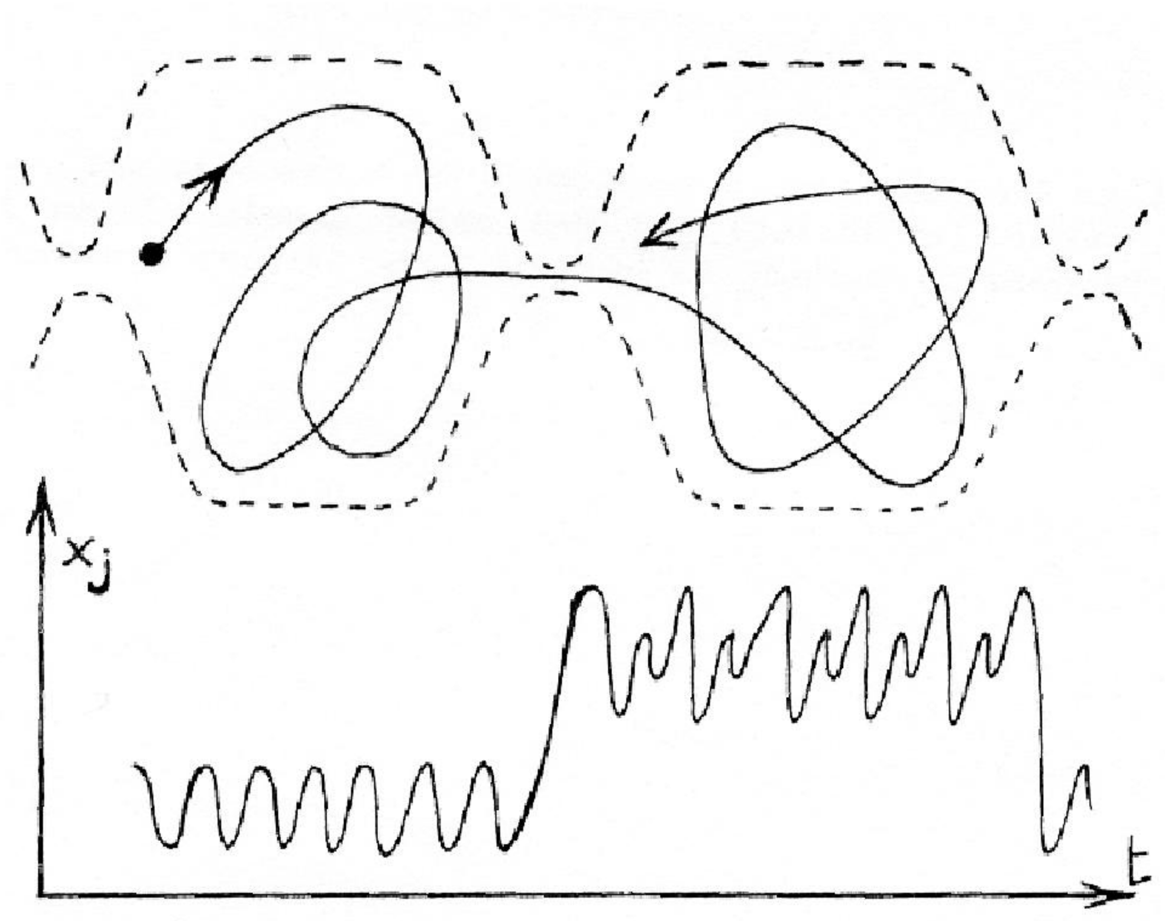
Important Tools

- Mechanical systems with symmetry; conserved quantities and reduction.
- For chaotic regimes of motion, the phase space has structures mediating transport.
- Theory of **tube dynamics** developed to study the motion of certain Jupiter-family comets (Koon, Lo, Marsden, SDR).
- Use the theory (Rom-Kedar, Wiggins, Haller,...) as well as the MANGEN software for **lobe dynamics** computations developed by Francois Lekien.
- Transport calculations for Mars' impact ejecta, comets, Kuiper-belt objects, etc.

Transport Theory

- ***Chaotic dynamics***
  $\Longrightarrow$ ***statistical methods***

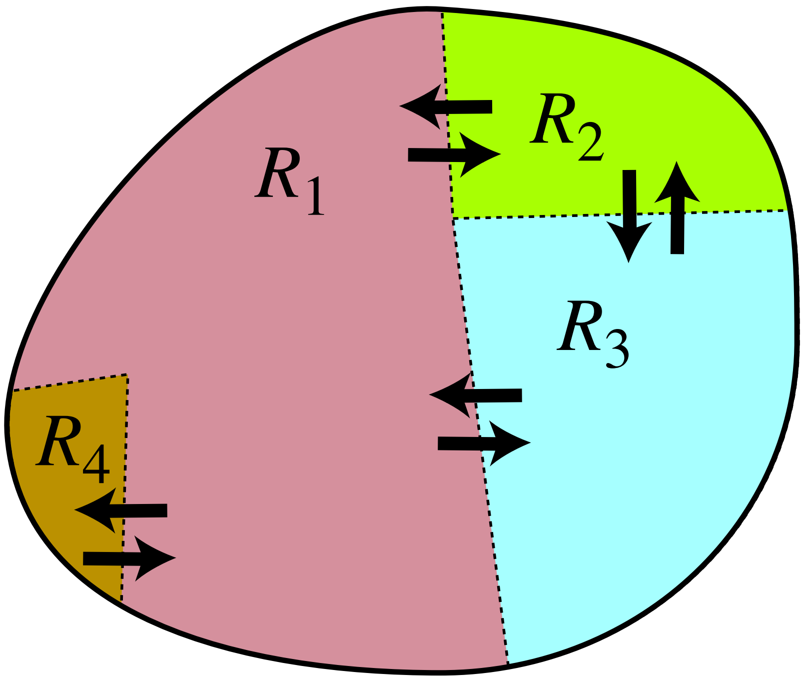
e.g., transport through "bottlenecks" in phase space; intermittency



Transport Theory

Ensembles of phase space trajectories

- Divide phase space into regions appropriately.
- How long to move from one region to another?
- Determine average transition rates.

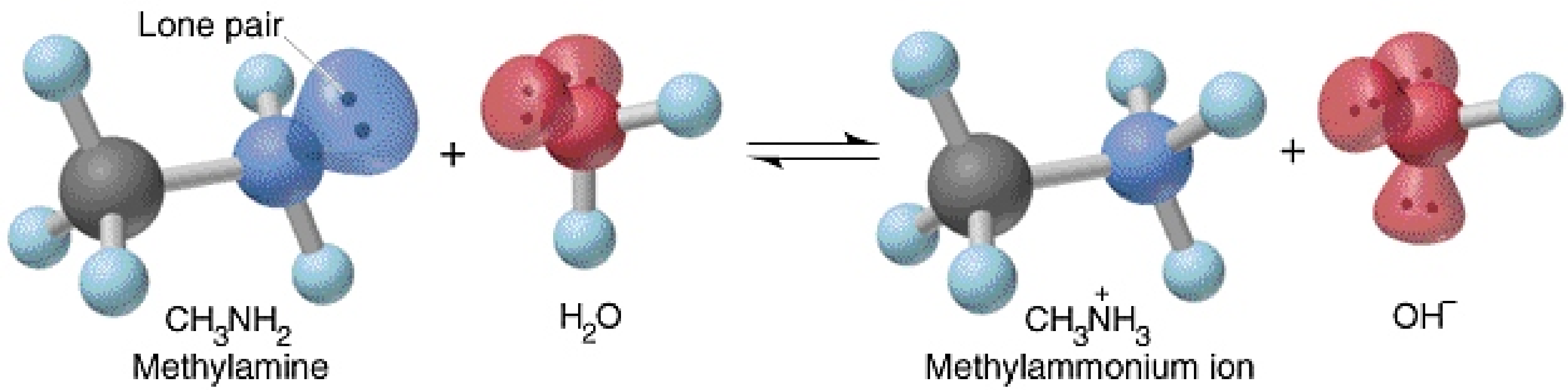


Boundaries between regions are "partial barriers" to transport.

Transport Theory

- ***Applications:***

- Geophysical fluid dynamics
- Chemical reaction rates



Transport Theory

- Comet and asteroid transport rates between appropriately defined regions; rates/probabilities of collision with a planet.

Insert movie of moon formation collision

Transport Theory

Transport in the solar system

- For minor bodies of interest
  - e.g., comets, Kuiper-belt objects, asteroids
- **Identify phase space objects** governing transport
- Model N-body system as restricted 3-body systems
- Assumption: Only one 3-body interaction dominates at a time
- e.g., comet-sun-P_1-P_2 system modeled as comet-sun-P_1 and comet-sun-P_2

Transport Theory

Insert pages from Marsden pres.

Transport Theory

Insert pages from Marsden pres.

Motion within Energy Shell

- For fixed μ, an energy shell (or energy manifold) of energy ε is

$$\mathcal{M}(\mu, \varepsilon) = \{(x, y, \dot{x}, \dot{y}) \mid E(x, y, \dot{x}, \dot{y}) = \varepsilon\}.$$

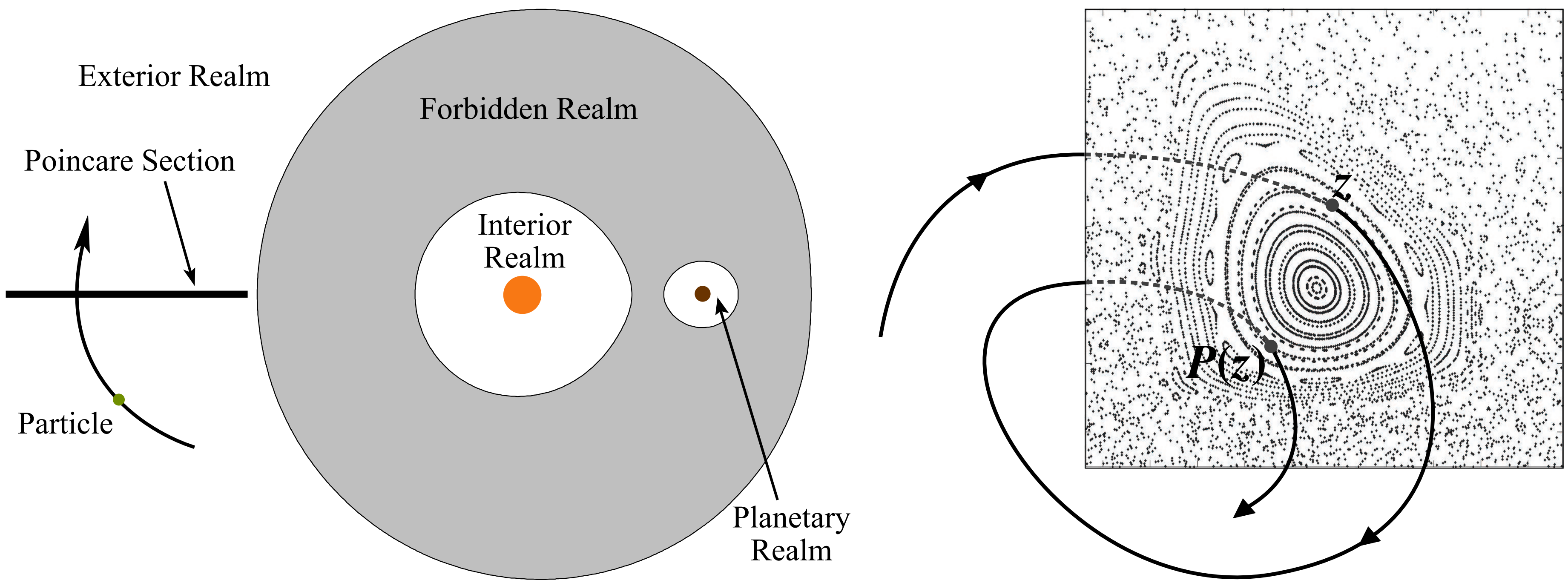
The $\mathcal{M}(\mu, \varepsilon)$ are 3-dimensional surfaces foliating the 4-dimensional phase space.

Poincaré Surface-of-Section

- Study **Poincaré surface of section** at fixed energy ε:

$$\Sigma_{(\mu,\varepsilon)} = \{(x, \dot{x}) | y = 0, \dot{y} = f(x, \dot{x}; \mu, \varepsilon) > 0\}$$

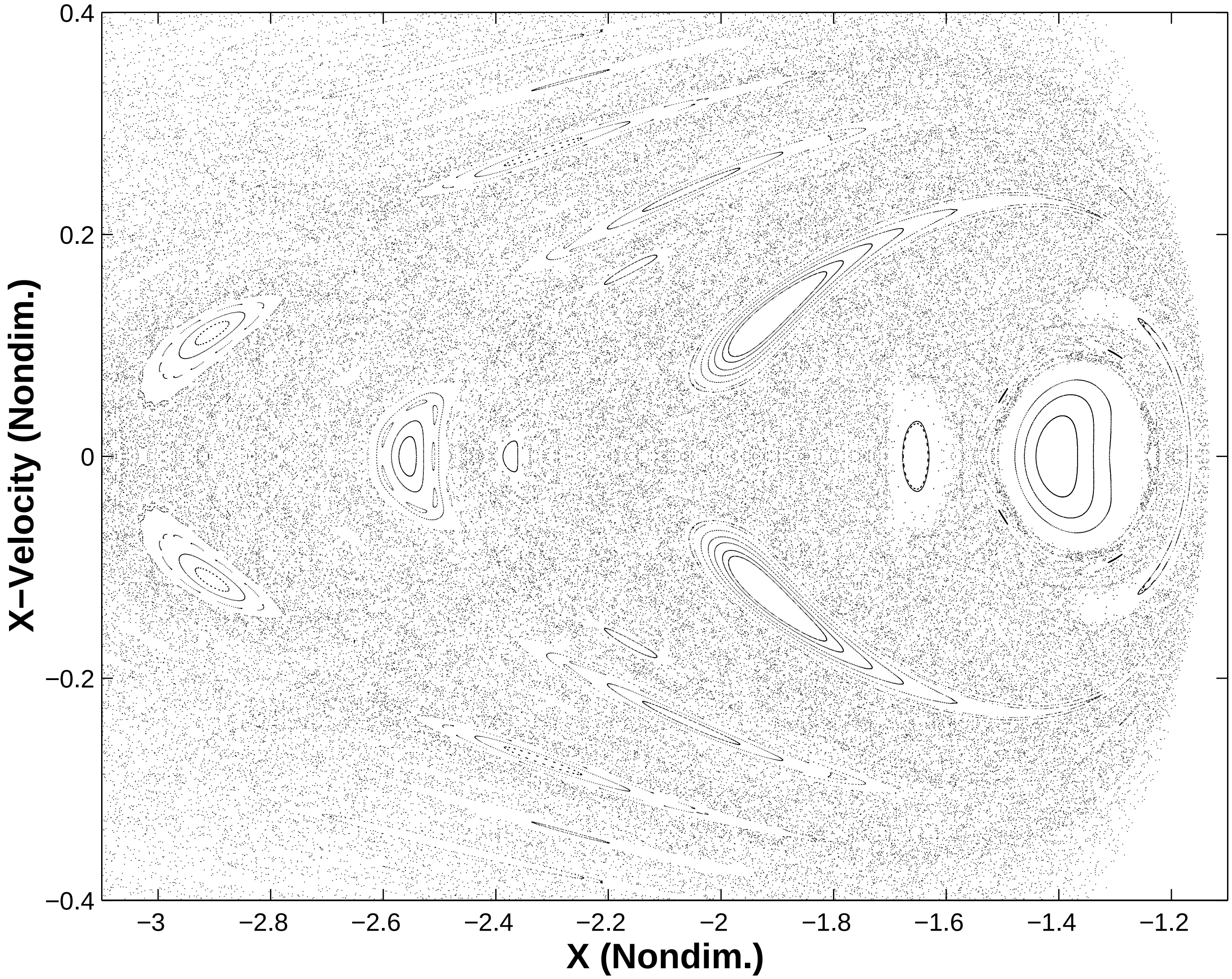
reducing the system to an area preserving map on the plane.



Poincaré surface-of-section and map P

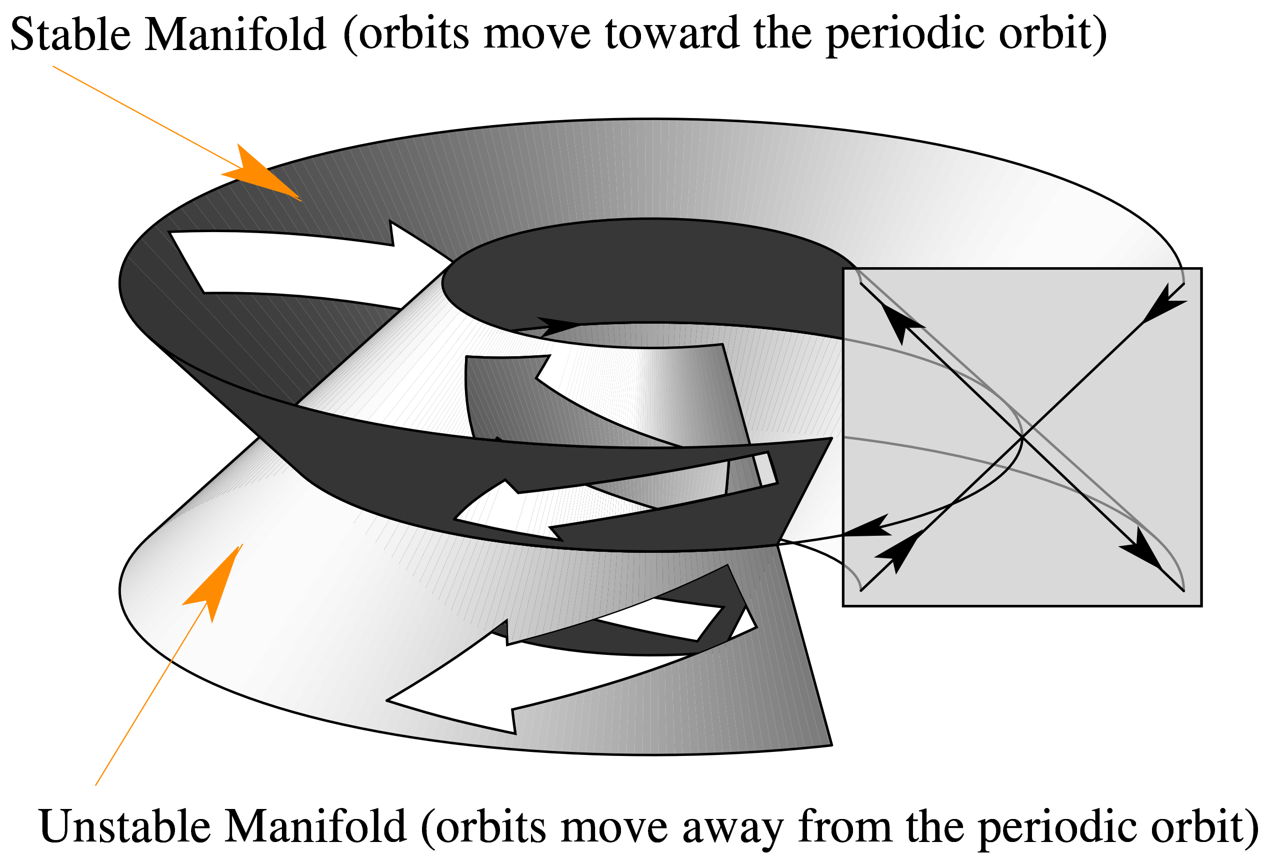
Chaotic Sea

- The energy shell has regular (KAM tori) and irregular components. Large connected irregular component, the **"chaotic sea."**



Transport in 3-Body Problem

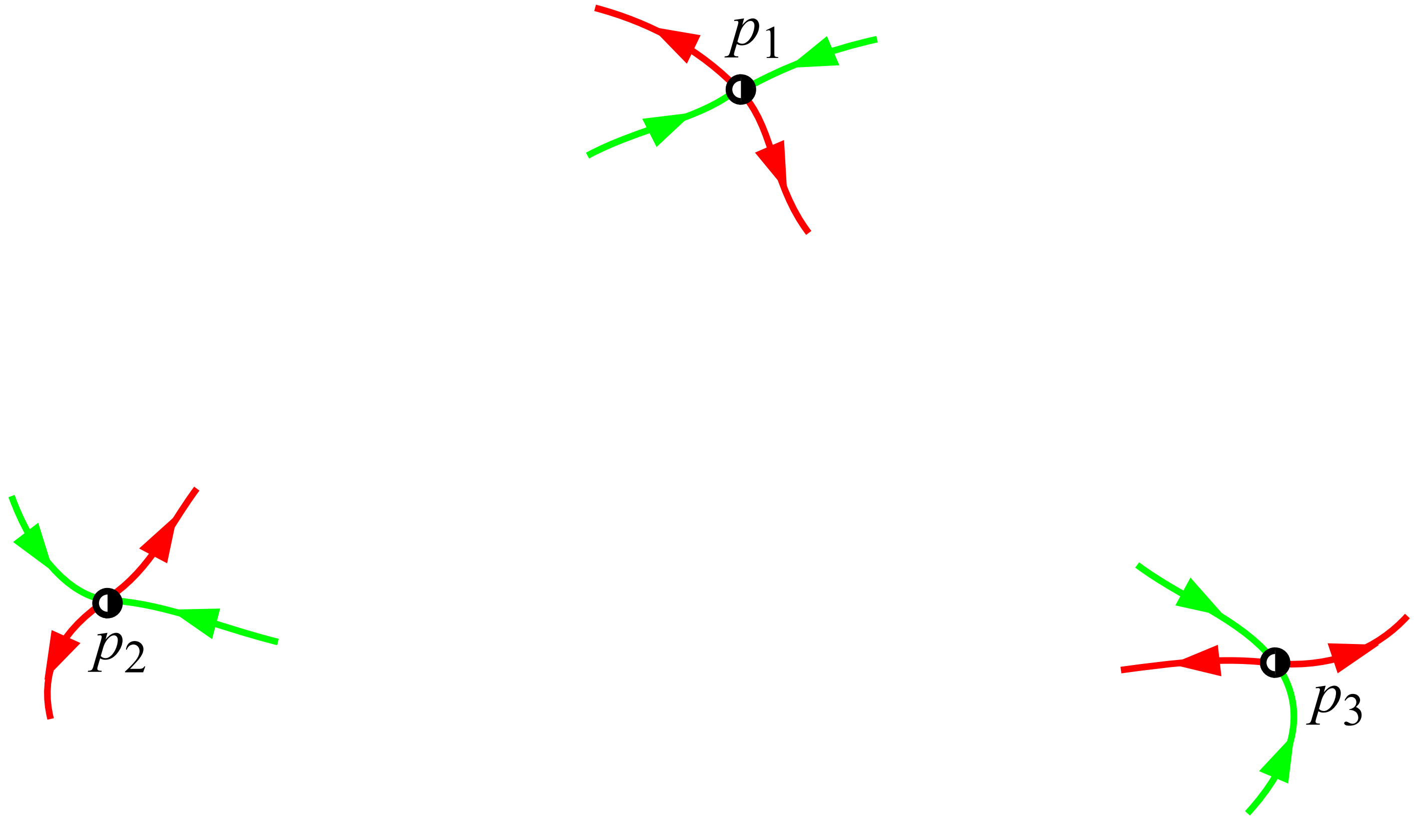
- Unstable resonances: Periodic orbits form a **dynamical "back-bone,"** via their unstable and stable manifolds.
- Physically, these manifolds correspond to orbits undergoing **repeated close encounters** with the smaller primary, e.g., Jupiter.



Unstable resonances and their manifolds.

Transport in 3-Body Problem

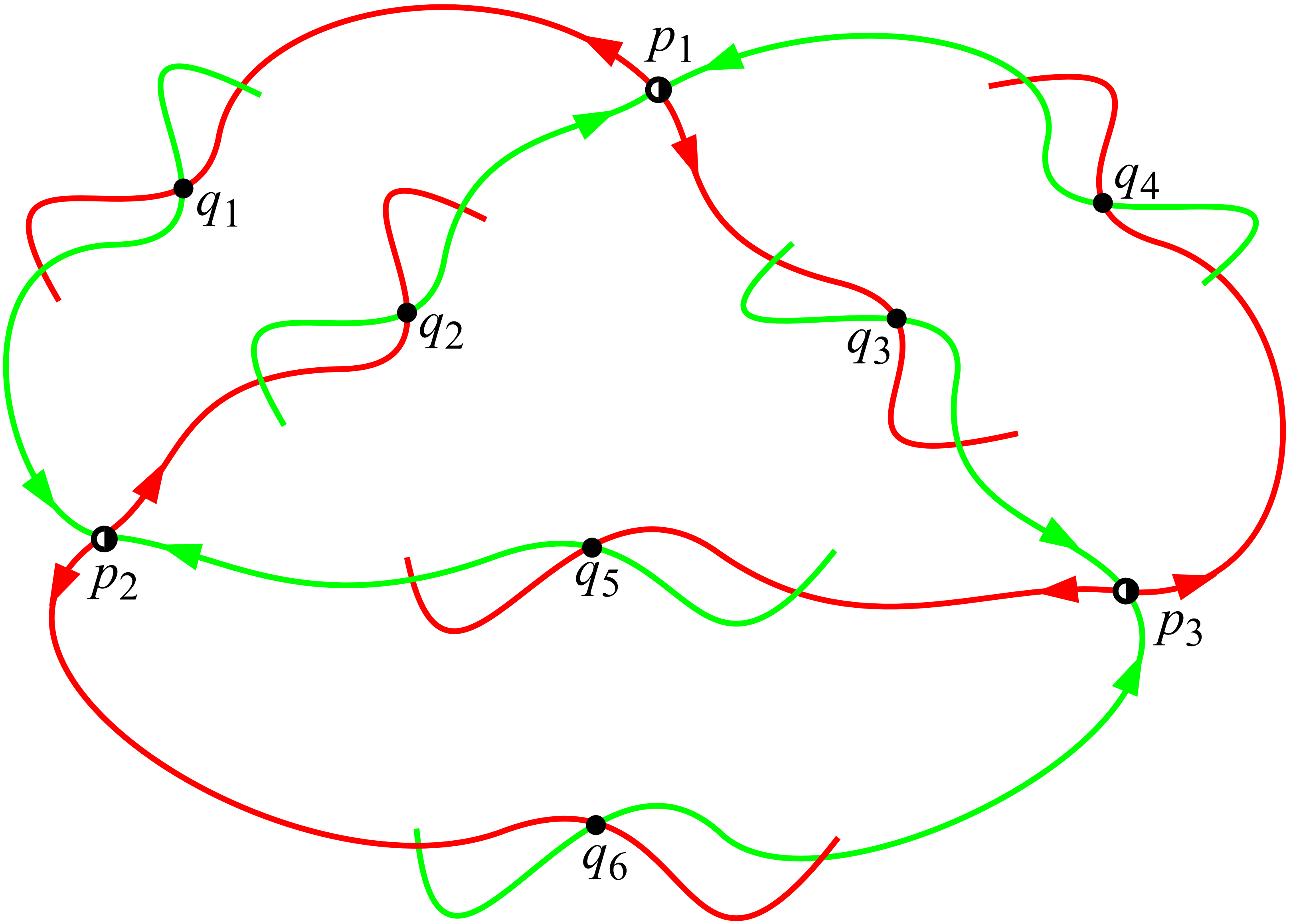
- On a Poincaré section, consider the **unstable and stable manifolds** of unstable periodic orbits



Unstable and stable manifolds in **red** and **green**, resp.

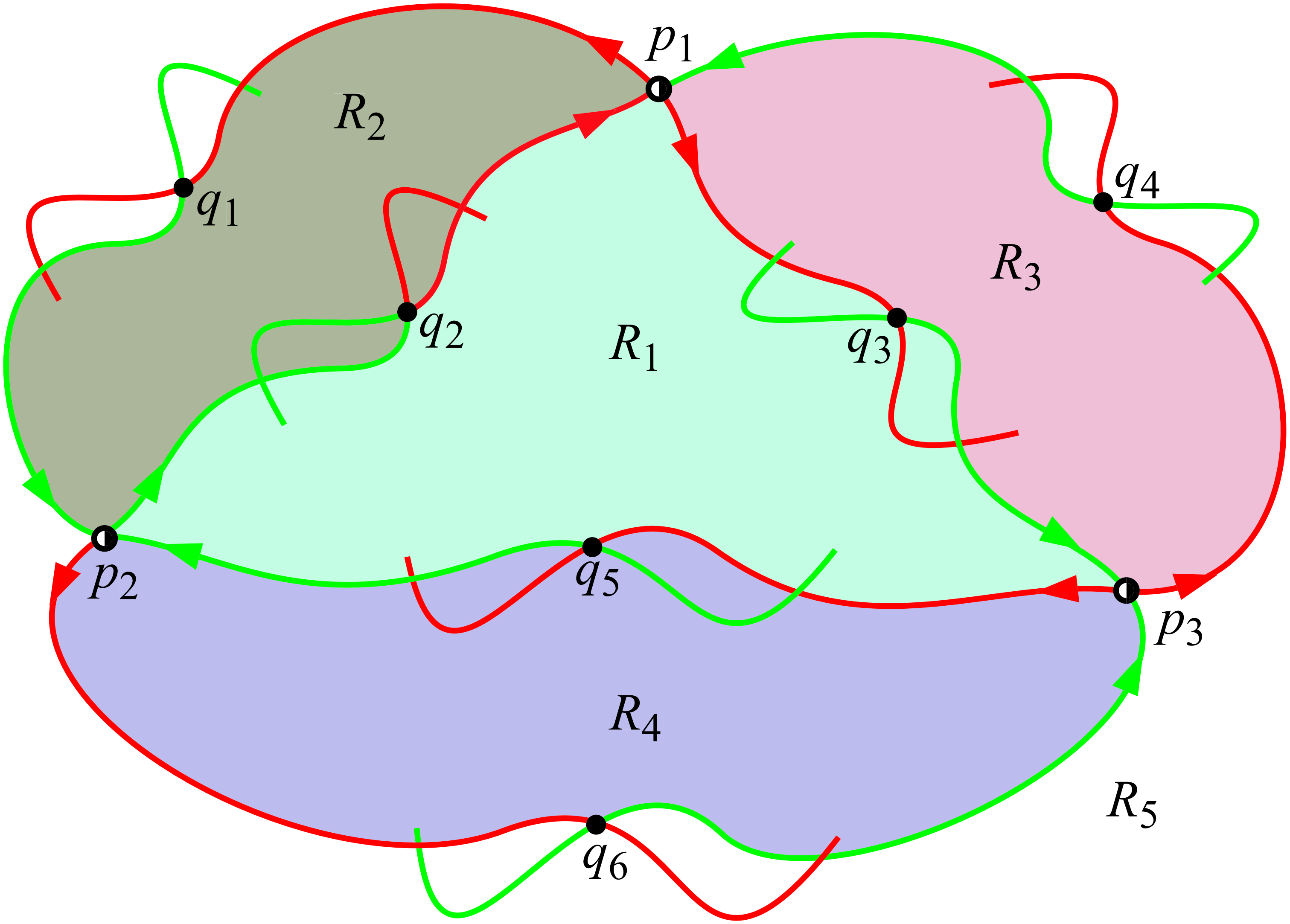
Transport in 3-Body Problem

- Intersection of unstable and stable manifolds define **boundaries**.



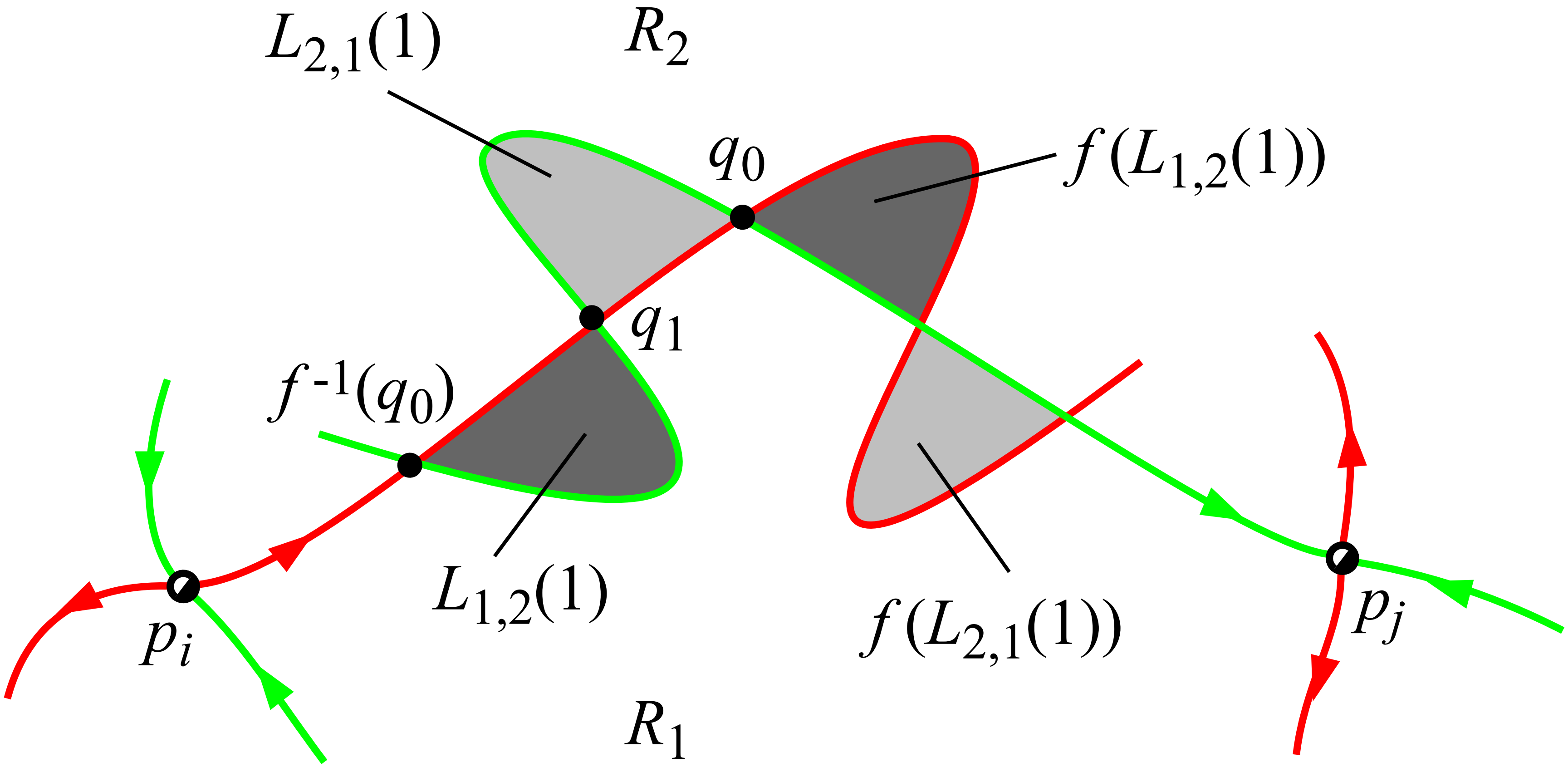
Transport in 3-Body Problem

- These boundaries divide the phase space into **regions**.



Lobe Dynamics

- Transport between regions is computed via **lobe dynamics.**

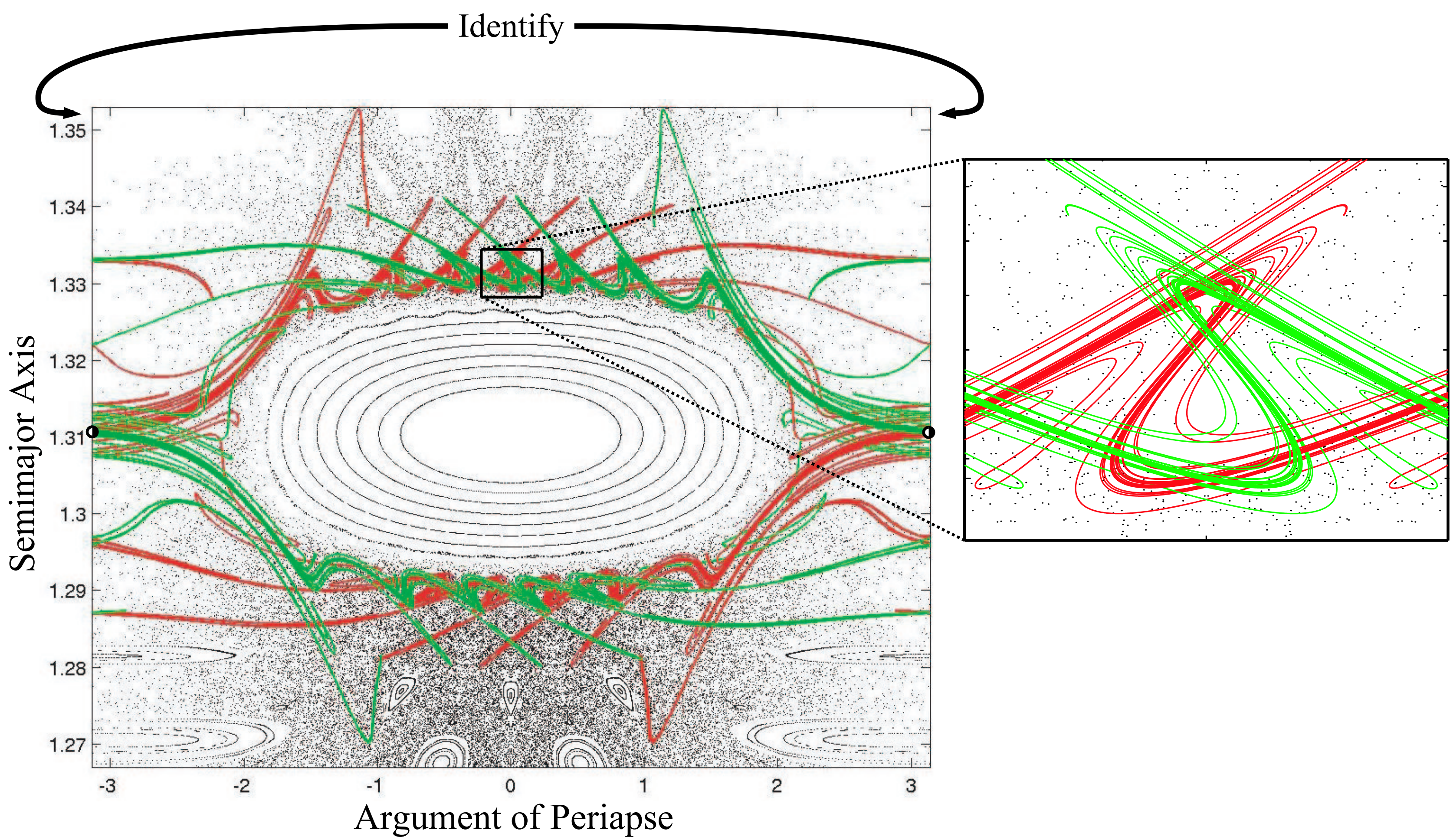


Dynamical Astronomy

- Compute transport between regions, e.g., transport between mean motion resonances, rates of ejecta escape from a planet, etc.
- Some questions of interest
  - How probable is a Shoemaker-Levy 9-type collision with Jupiter? Or an asteroid collision with Earth (e.g., KT impact 65 Ma)?
  - How likely is a transition from outside a planet's orbit to inside (e.g., the dance of comet Oterma with Jupiter)?
- Harder questions
  - How does impact ejecta get from Mars to Earth?
  - How does an SKBO become a comet or an Oort Cloud comet?
  - Find features common to all exo-solar planetary systems?

Movement btwn Resonances

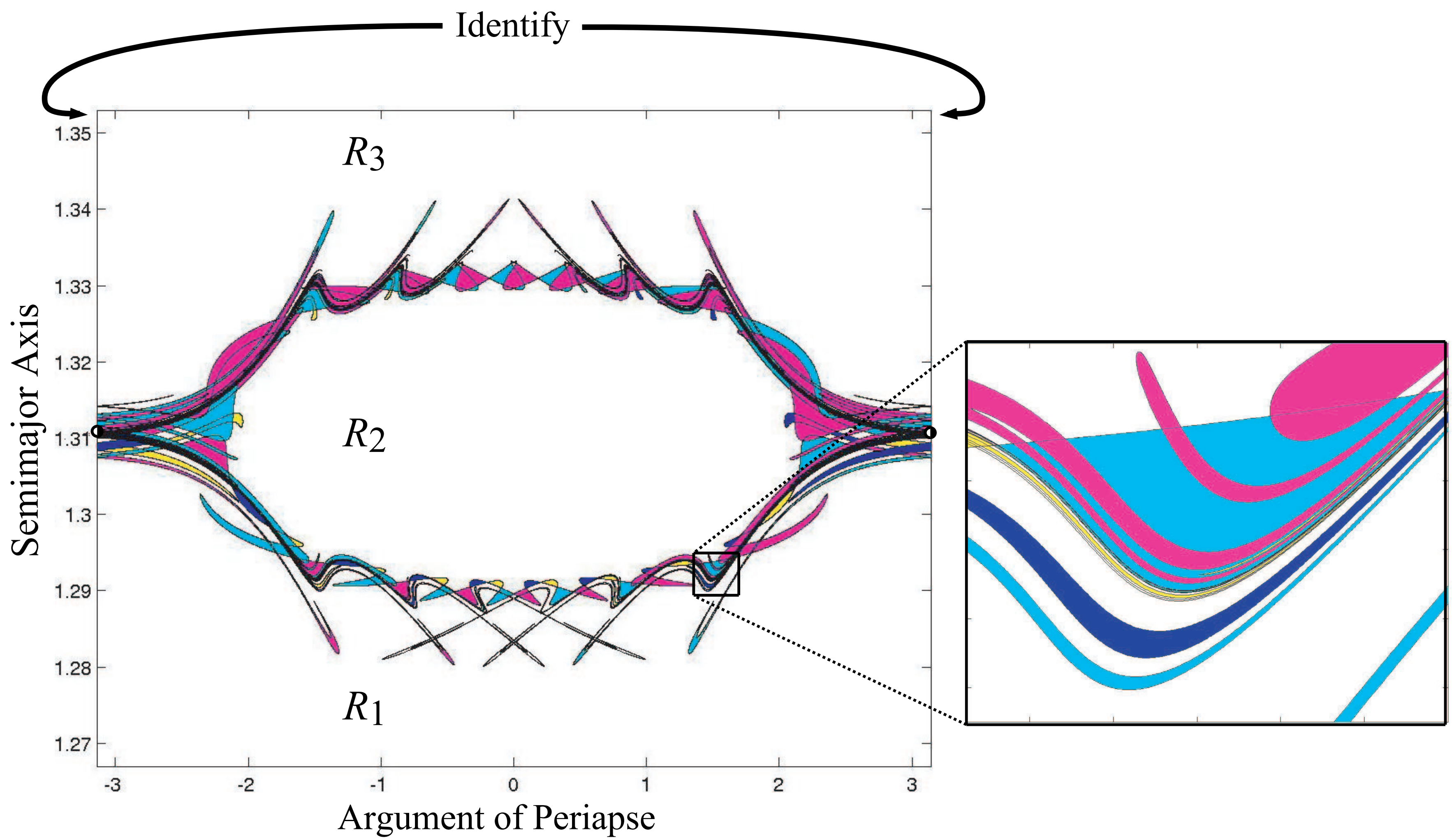
- We can compute manifolds which naturally divide the phase space into **resonance regions.**



Unstable and stable manifolds in **red** and **green**, resp.

Movement btwn Resonances

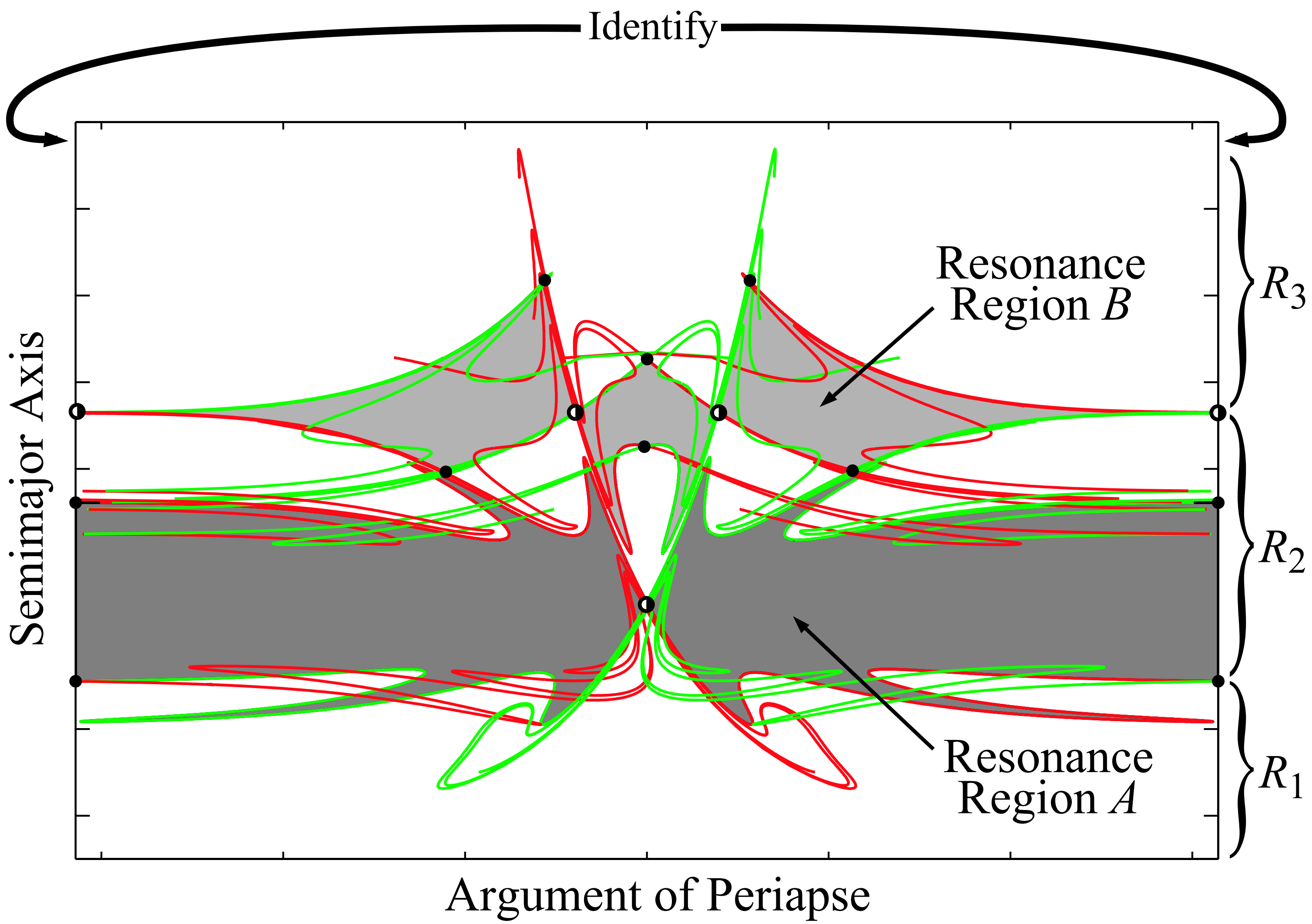
- Transport and mixing between regions can be computed.



Four sequences of color coded lobes are shown.

Movement btwn Resonances

- Transport and mixing between several resonances can be computed.



Oceanic Interlude

- The software used to compute transport by lobe dynamics, namely **MANGEN**, comes from a study of ocean dynamics.
- Interesting: **there are analogs of navigating by invariant manifolds in the ocean.**
- Adaptive Ocean Sampling Network (AOSN-II)
  - **Princeton:** Naomi Leonard, Clancy Rowley, Eddie Forelli, Ralf Bachmayer, ...
  - **Caltech:** Chad Couliette, Francois Lekien, Jerry Marsden, Shawn Shadden
  - **MIT:** George Haller

Oceanic Interlude

Insert movie of parcels

Oceanic Interlude

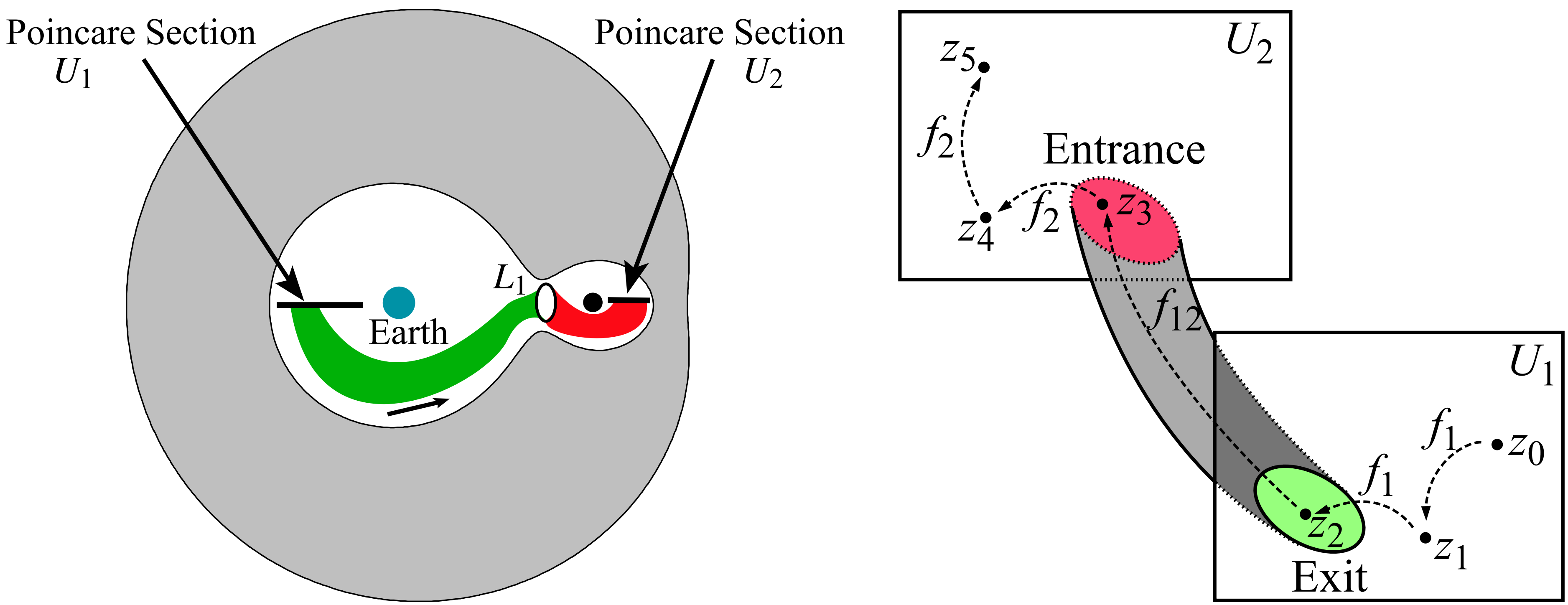
Insert movie of parcels w/ mfds

Tube Dynamics

- ***Back to the 3-body problem...***
- ***Must also consider tube dynamics!***
  - Tubes in the energy surface lead toward and away from bottlenecks.
    - Conley, McGehee (1960s)
    - Koon, Lo, Marsden, SDR (2000s)

Tube Dynamics

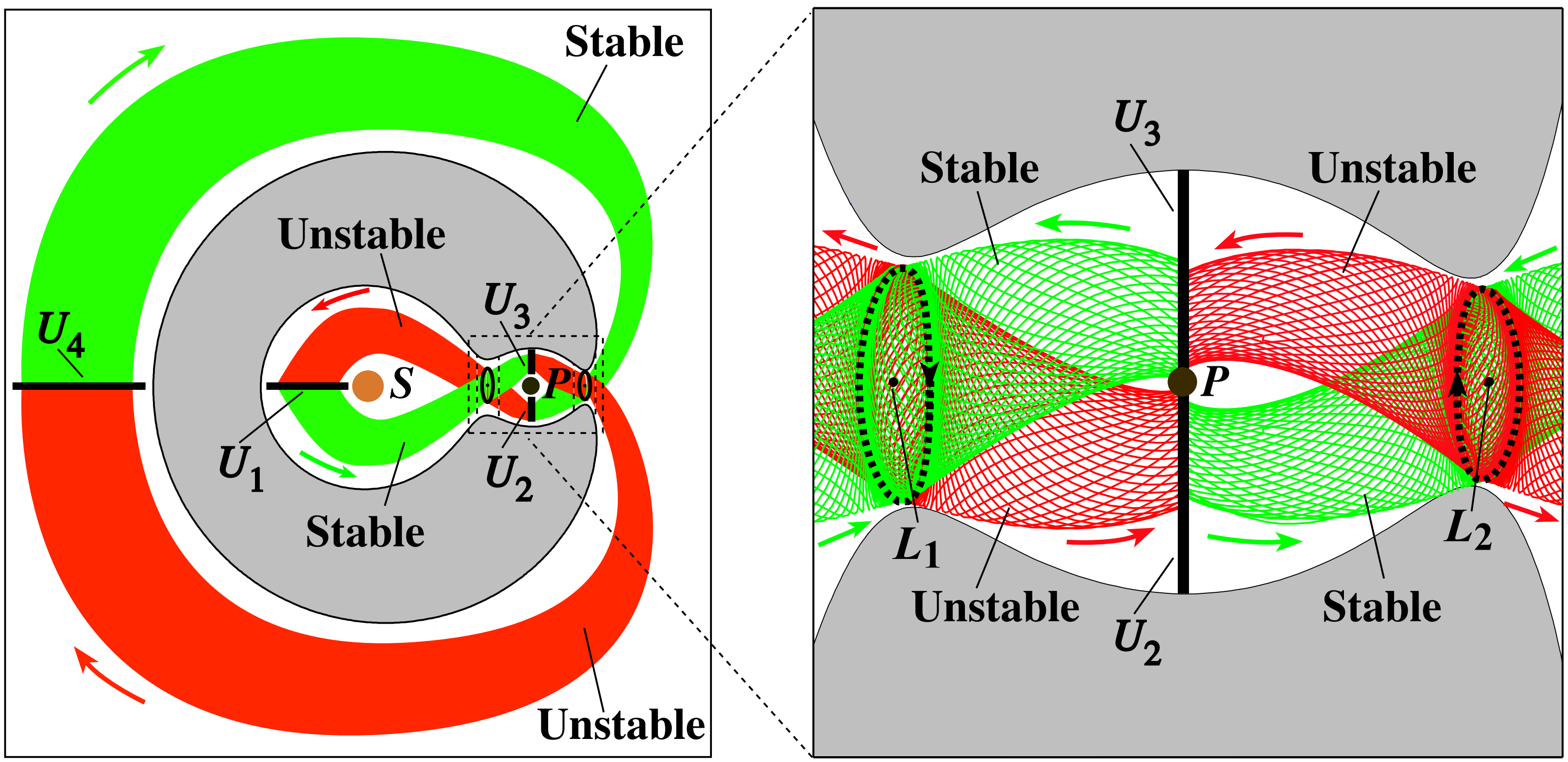
- For example, points reach the **exit** in U_1 and are transported via a tube to the **entrance** of U_2.



Tube dynamics: going from one Poincaré section to another.

Tube Dynamics

- Poincaré sections in different realms (U_1 through U_4) are linked by phase space tubes. The projection of the tubes on the configuration space appear as strips.



Unstable and stable manifolds in **red** and **green**, resp.

Resonances and Tubes

- ***Resonances and tubes are linked***
  - It has been observed that the tubes of capture orbits are coming from certain resonances.
    - Koon, Lo, Marsden, SDR [2001]

Jupiter Family Comets

- ***Jupiter Family Comets***
  - A physical example of the link between resonances and tubes
  - We consider the historical record of the comet Oterma from 1910 to 1980
    - first in an inertial frame
    - then in a rotating frame
    - a special case of pattern evocation
  - similar pictures exist for many other comets

Jupiter Family Comets

- Rapid transition: outside to inside Jupiter's orbit.
  - Captured temporarily by Jupiter during transition.
  - Exterior (2:3 resonance) to interior (3:2 resonance).

Viewed in Rotating Frame

- Oterma's orbit in rotating frame with some invariant manifolds of the 3-body problem superimposed.

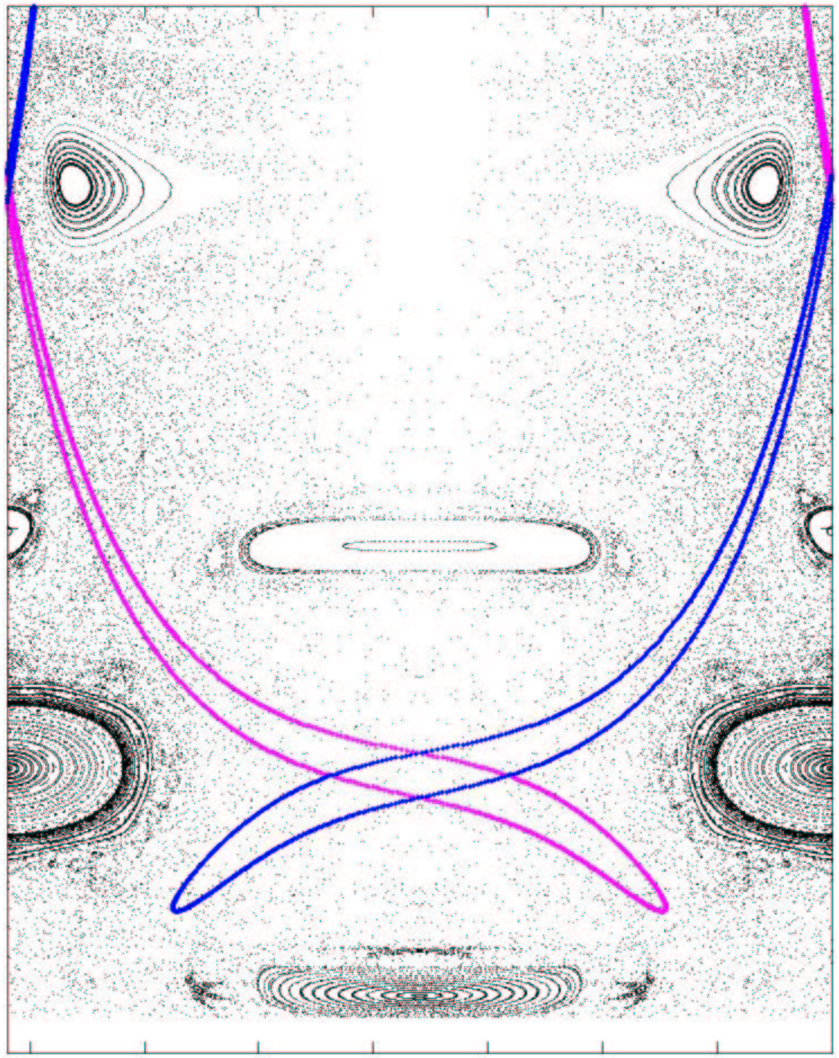
oterma-rot.qt

Viewed in Inertial Frame

oterma-iner.qt

Resonances and Tubes

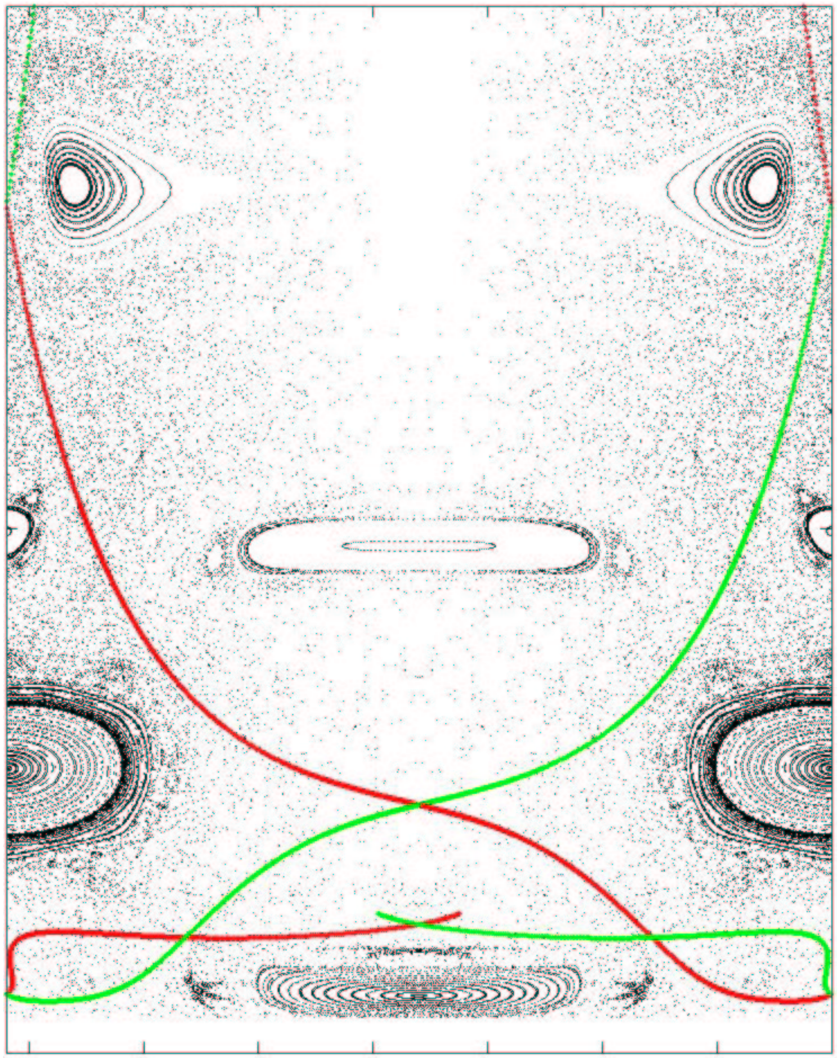
- Poincaré section: tube cross-sections are closed curves



Particles inside curves move toward or away from Jupiter

Resonances and Tubes

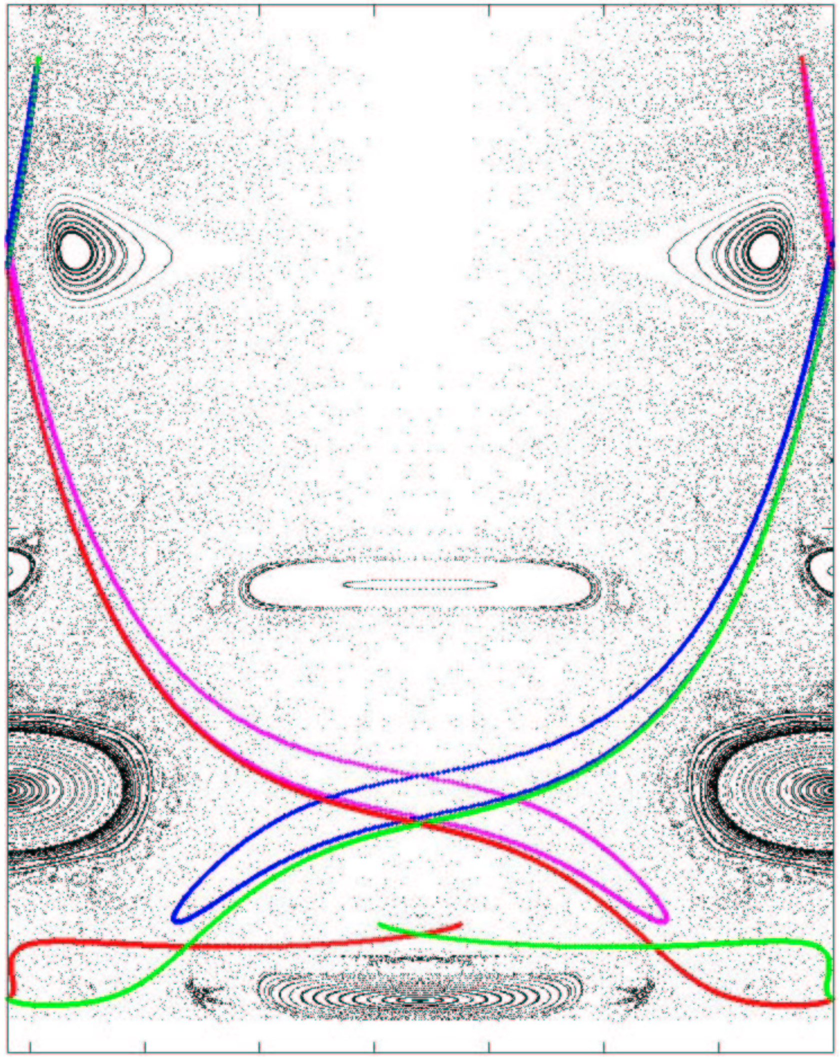
- Same Poincaré section: a resonance region is plotted



2:3 exterior resonance region

Resonances and Tubes

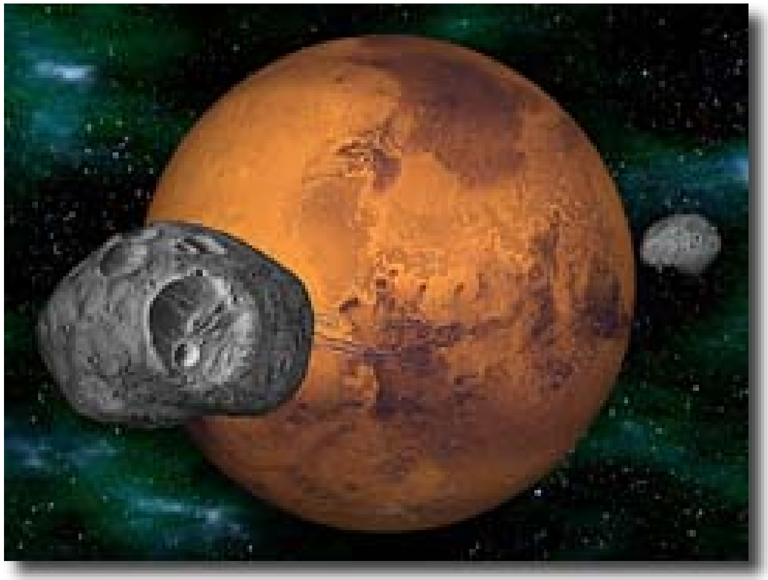
- Regions of overlap occur $\longrightarrow$ **complex dynamics!**



Regions of overlap occur

Escape Rates

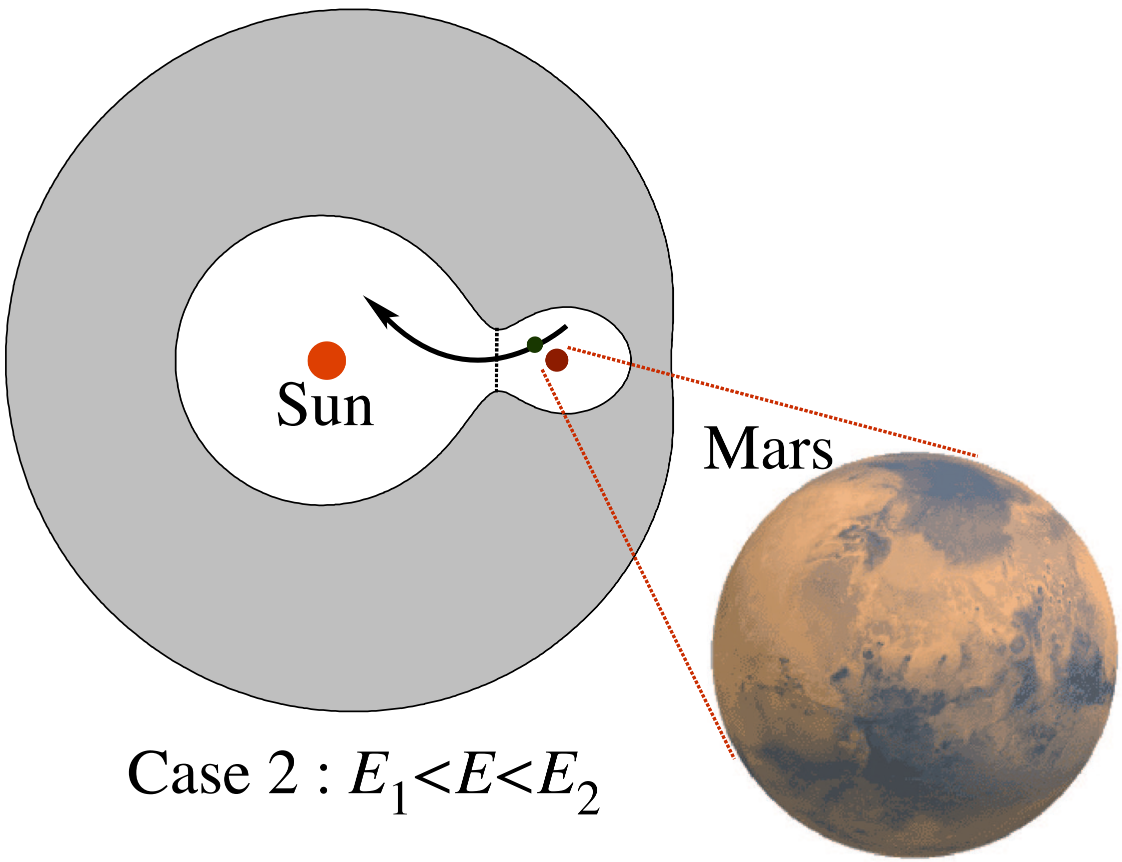
- ***Applications to dynamical astronomy***
  - One can compute the rate of escape of particles temporarily captured by Mars, e.g. asteroids or impact ejecta liberated from the Martian surface.
    - Jaffé, SDR, Lo, Marsden, Farrelly, and Uzer [2002]



Mars with temporarily captured asteroids.

Escape Rates

- Consider a particle at an energy such that it can escape sunward. Using a **statistical approach** used in transition state theory (developed by chemists), the rate of escape can be estimated.

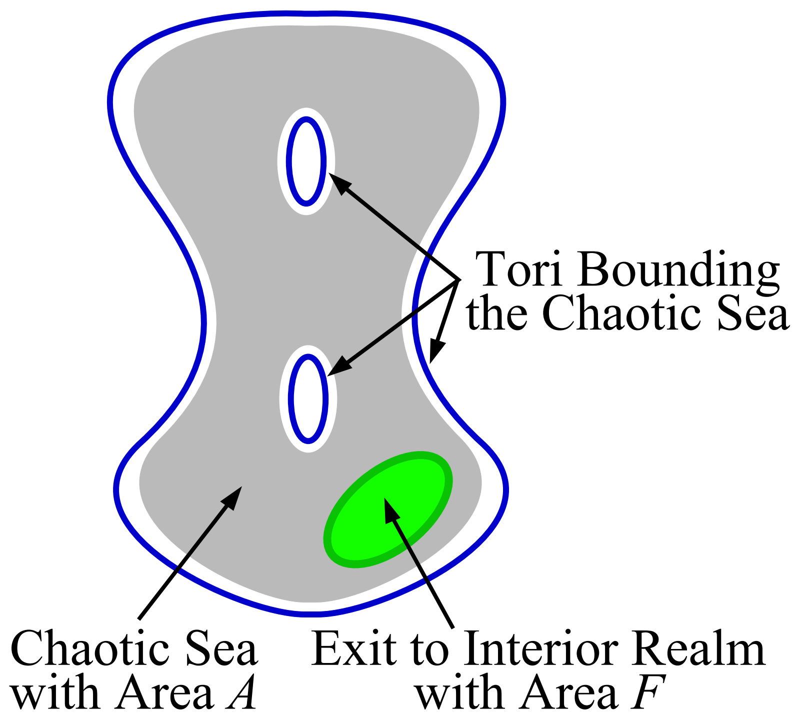


Escape Rates

- **Mixing assumption:** all asteroids in the chaotic sea surrounding Mars are **equally likely to escape**.

  Escape rate $= -\log(1-p)$, where

$$p = \frac{\text{Area of exit sunward}}{\text{Area of chaotic sea}}$$

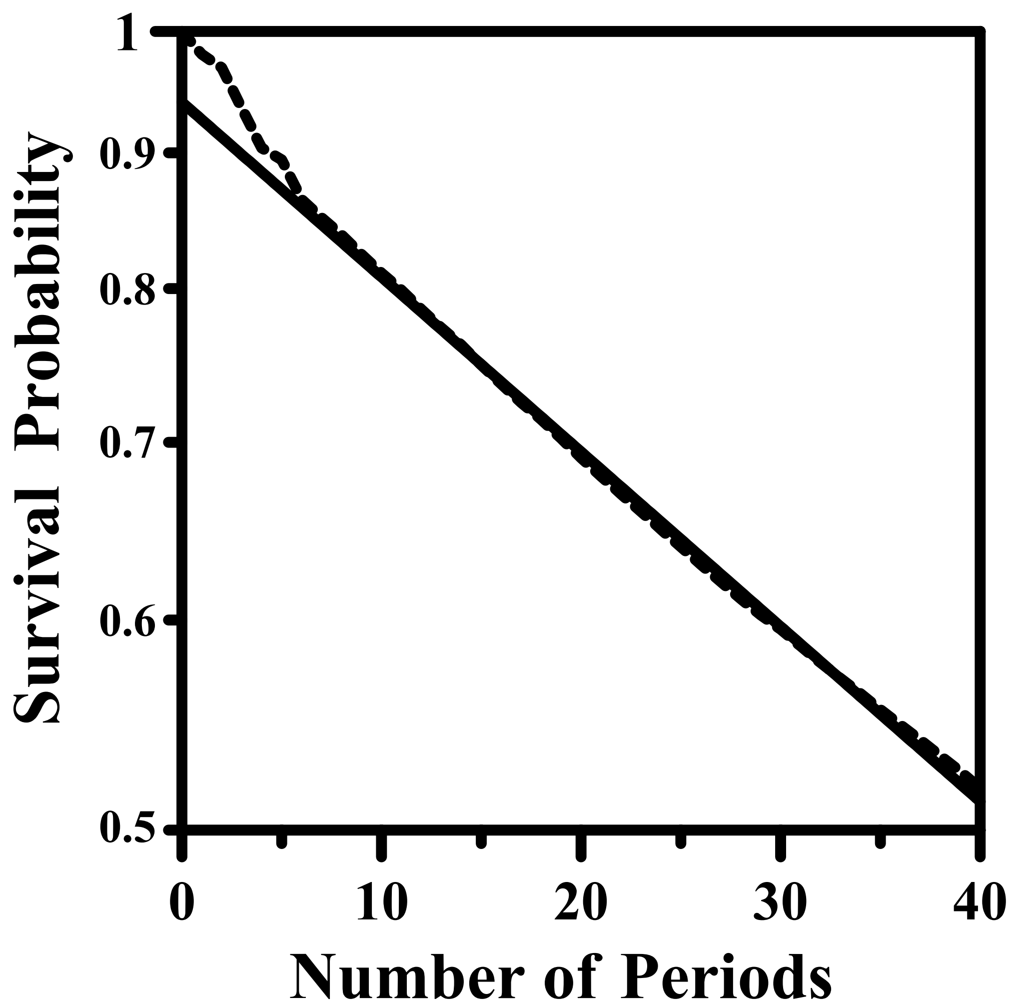


Escape Rates

- This is a particularly simple situation ("Markovian")
- Compare this rate with one obtained from a Monte Carlo simulations of 107,000 particles at randomly selected initial conditions at the same energy.

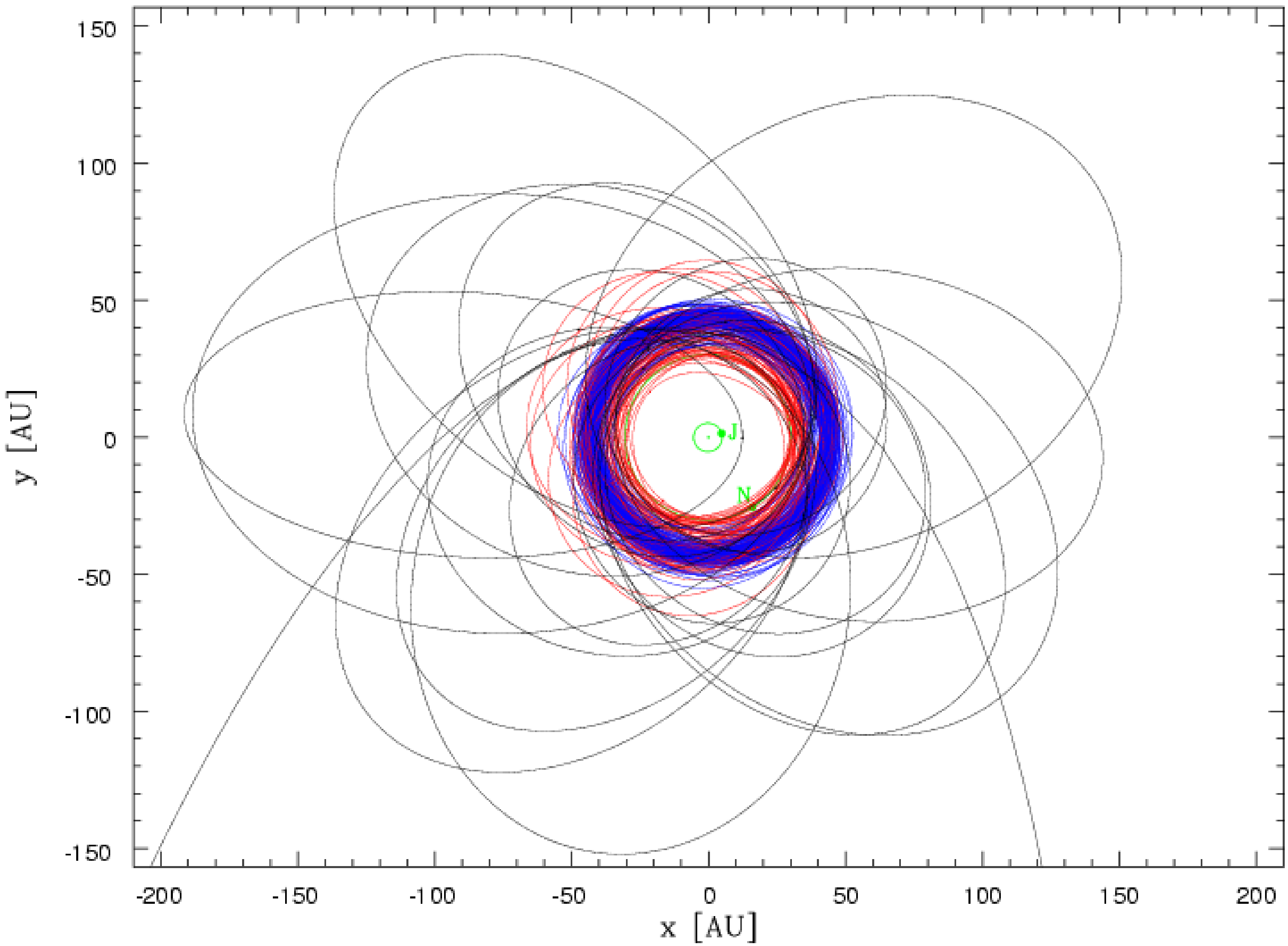
Escape Rates

- Theory and numerical simulations agree well
  - Monte Carlo simulation (dashed) and theory (solid)



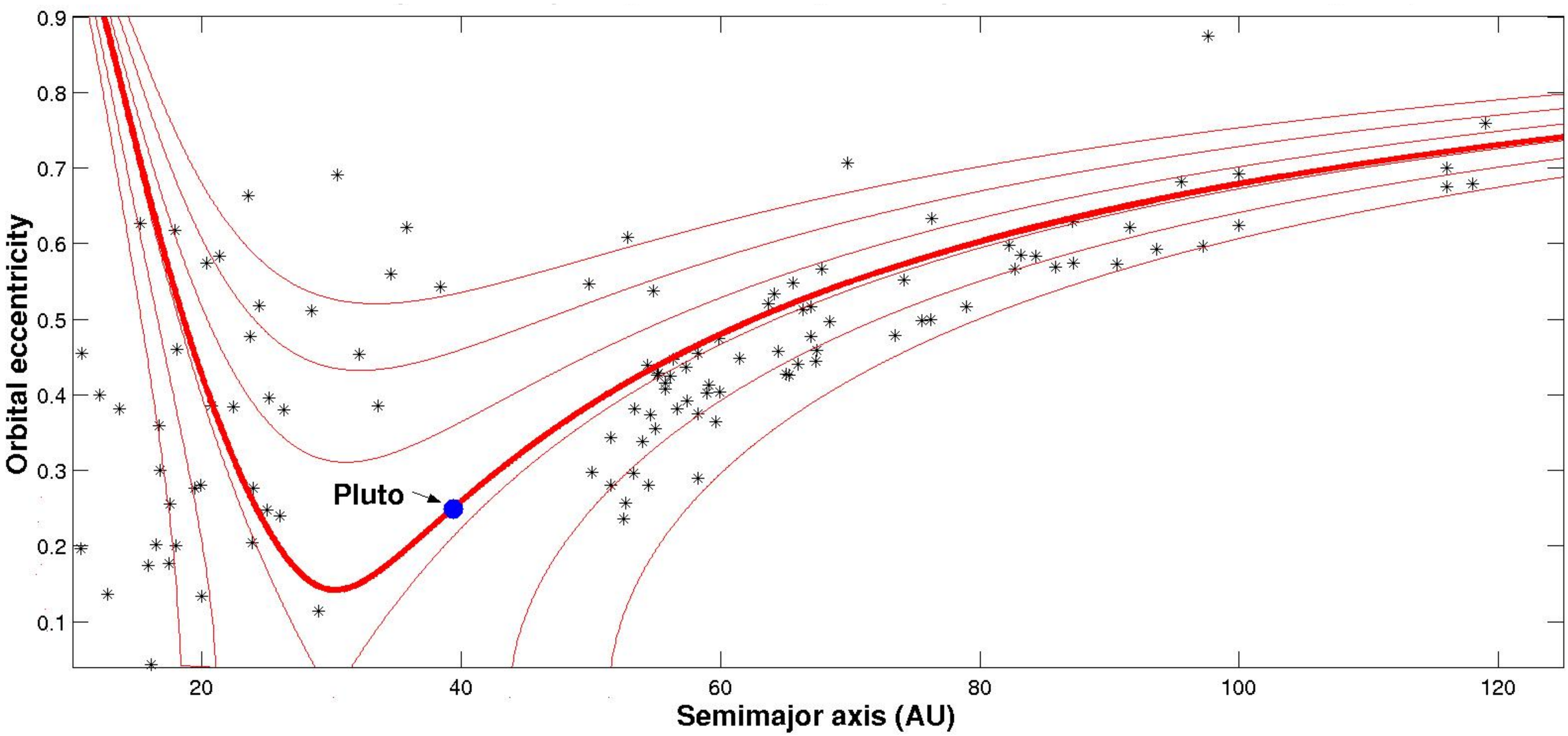
Scattered Kuiper Belt Objects

- Some scattered Kuiper Belt Objects (SKBOs) in inertial space.



Scattered Kuiper Belt Objects

- Current SKBO locations in black, with some approximate curves of constant energy in the Sun-Neptune-SKBO in red.



Steady State Distribution

- If the planar, circular restricted three-body problem is approximately **ergodic,** then a statistical mechanics can be built (cf. ZhiGang [1999]).
- Recent work suggests there may be regions of the energy shell for which the motion is nearly ergodic, in particular the "chaotic sea" (Jaffé et al. [2002]).
- This suggests we compute the **steady state distribution** of some observable for particles in the chaotic sea; a simple method for obtaining the likely locations of any particles within it.

Steady State Distribution

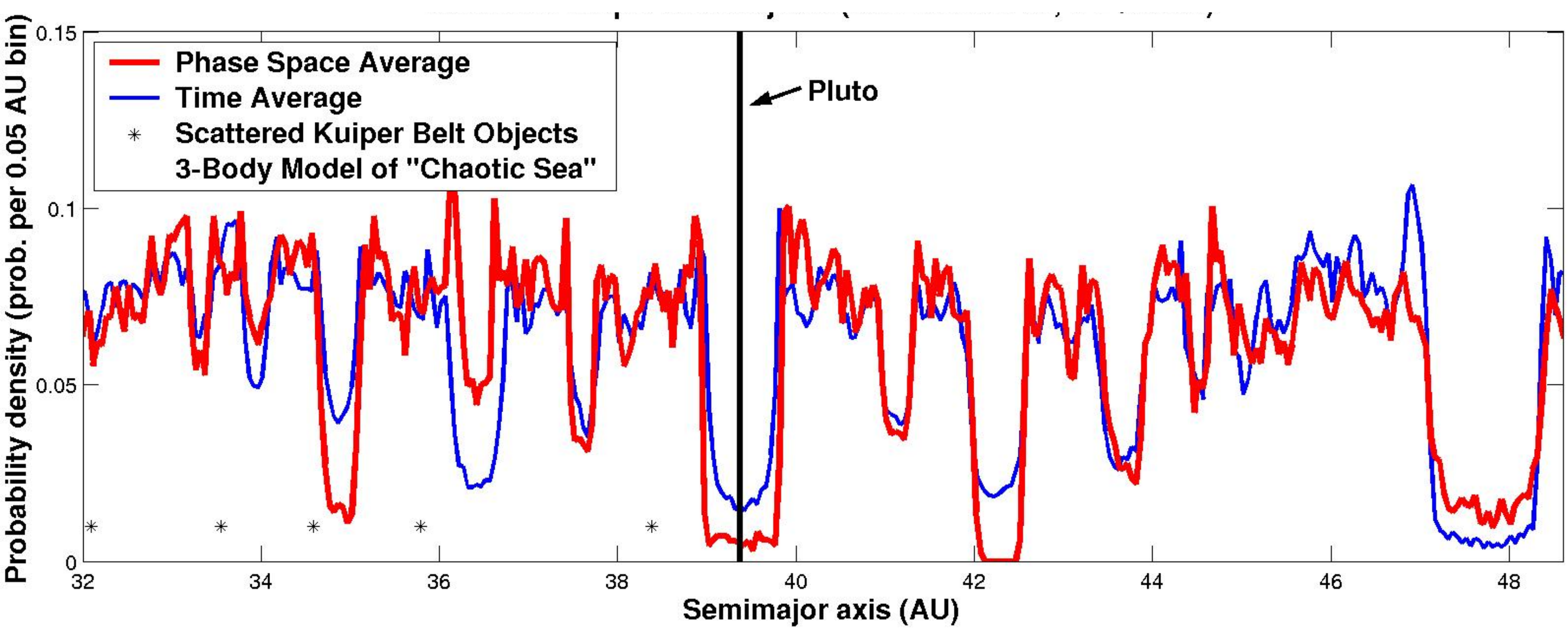
- Assuming ergodicity,

$$\lim_{t\to\infty}\frac{1}{t}\int_0^t A(x,y,p_x,p_y)d\tau = \int A(x,y,p_x,p_y)\,\frac{C}{|\frac{\partial H}{\partial p_y}|}dp_x dx dy,$$

  where $A(x,y,p_x,p_y)$ is any physical observable (e.g., semimajor axis), one can finds that the density function, $\rho(x,p_x)$, on the surface-of-section, $\Sigma_{(\mu,\varepsilon)}$, is constant.
- We can determine the steady state distribution of semi-major axes; define $N(a)da$ as the number of particles falling into $a \to a+da$ on the surface-of-section, $\Sigma_{(\mu,\varepsilon)}$.

Steady State Distribution

- SKBOs should be in regions of high density.



Selected References

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For papers, movies, etc., visit the websites:

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http://transport.caltech.edu/