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Invariant Manifolds, the Spatial 3-Body Problem and Space Mission Design

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Introduction

Theme

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- ***Current research importance***
  - Development of some NASA mission trajectories, such as the recently launched *Genesis Discovery Mission*, and the upcoming *Europa Orbiter Mission*

Introduction

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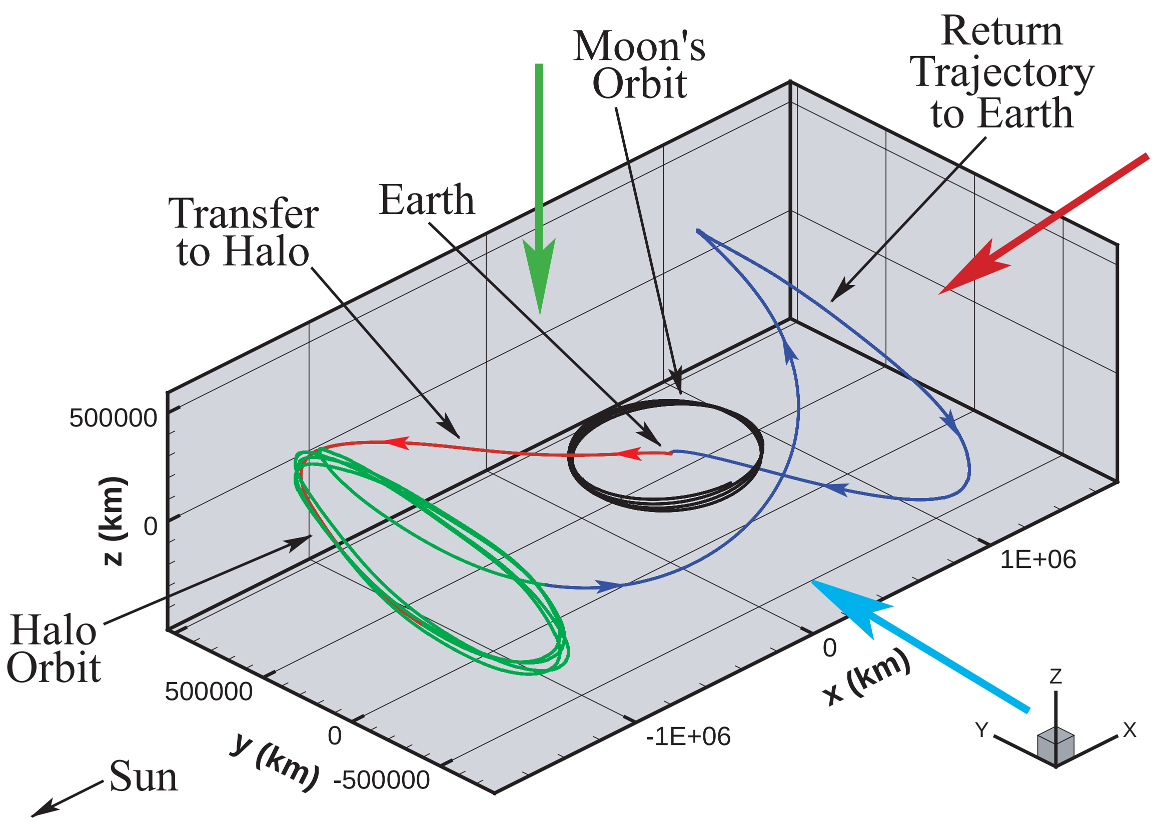
- ***Current research importance***
  - Development of some NASA mission trajectories, such as the recently launched *Genesis Discovery Mission*, and the upcoming *Europa Orbiter Mission*
  - Of current astrophysical interest for understanding the transport of solar system material (eg, how ejecta from Mars gets to Earth, etc.)

Genesis Discovery Mission

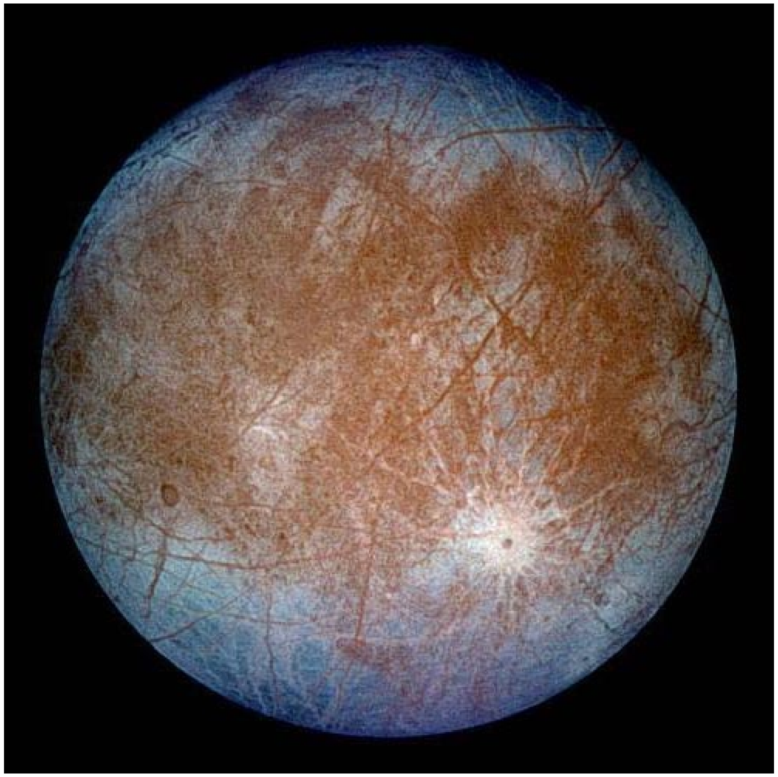
- Genesis will collect solar wind samples at the Sun-Earth L1 and return them to Earth.
- It was the first mission designed start to finish using dynamical systems theory.

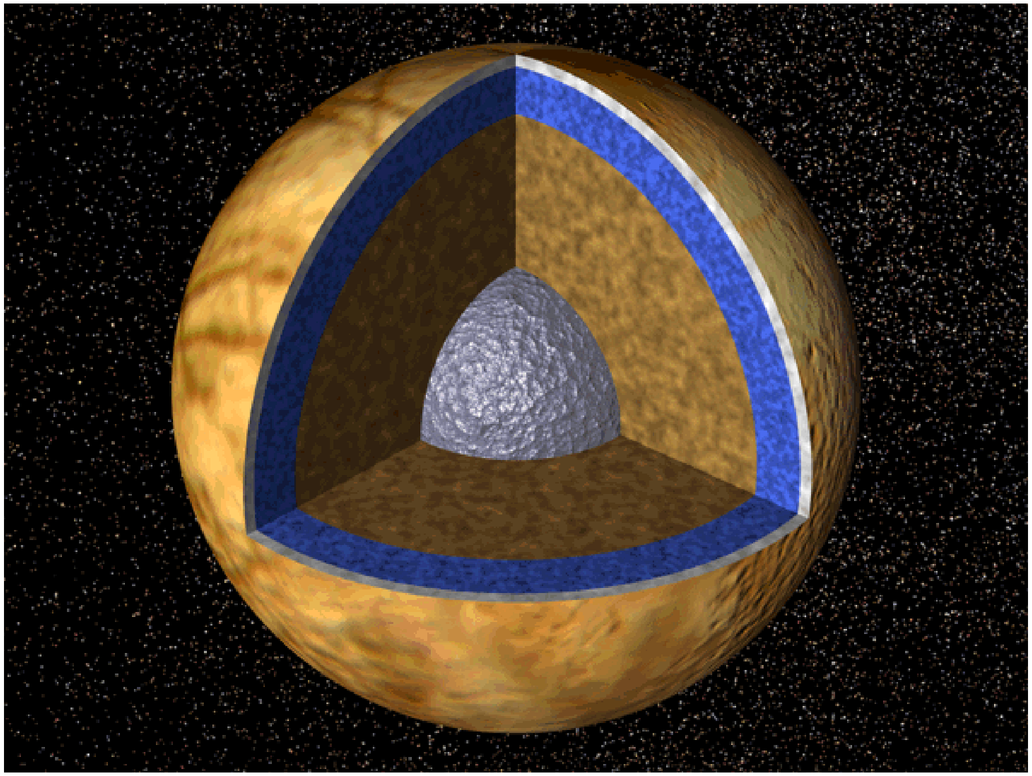


Europa Orbiter Mission

Oceans and life on Europa?

- Recently, there has been interest in sending a scientific spacecraft to orbit and study **Europa**.





Europa Orbiter Mission

Europa Mission

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- **Model:** Jupiter-Europa-Ganymede-spacecraft 4-body model considered as two 3-body models
  - Extend previous work from planar model to 3D

History of 3-Body Problem

Brief history:

- Founding of dynamical systems theory: Poincaré [1890]
- Special orbits: Conley [1963,1968], McGehee [1969]
- Invariant manifolds: Simó, Llibre, and Martinez [1985], Gómez, Jorba, Masdemont, and Simo [1991]
- Applied to space missions: Howell, Barden, and Lo [1997], Lo and Ross [1997,1998] Koon, Lo, Marsden, and Ross [2000], Gómez, Koon, Lo, Marsden, Masdemont, and Ross [2001]
- Using optimal control: Serban, Koon, Lo, Marsden, Petzold, Ross, and Wilson [2001]

Three-Body Problem

- ***Circular restricted 3-body problem***
  - the two primary bodies move in circles; the much smaller third body moves in the gravitational field of the primaries, without affecting them

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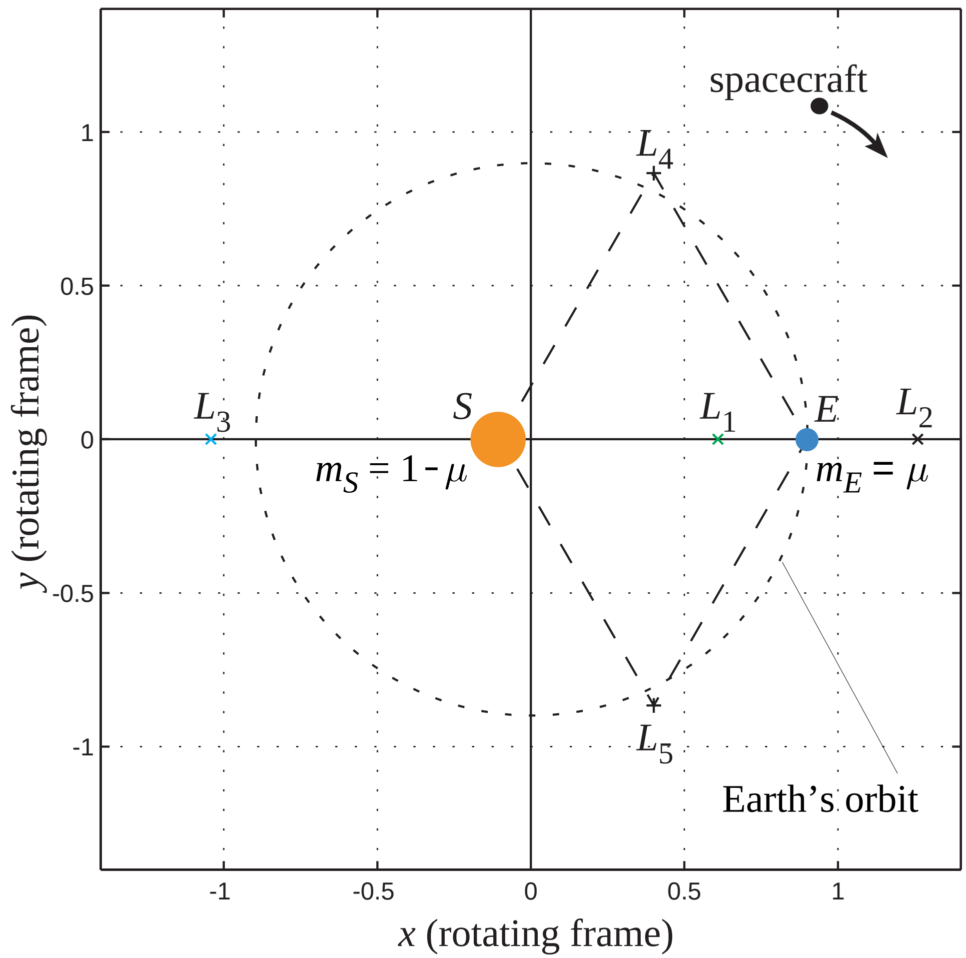
  - we consider the planar and spatial problems
  - there are five equilibrium points in the rotating frame, places of balance which generate interesting dynamics

Three-Body Problem

- 3 unstable points on line joining two main bodies – L_1, L_2, L_3
- 2 stable points at $\pm 60°$ along the circular orbit – L_4, L_5



Equilibrium points

Three-Body Problem

- orbits exist around L_1 and L_2; both periodic and quasi-periodic
  - Lyapunov, halo and Lissajous orbits
- one can draw the invariant manifolds assoicated to L_1 (and L_2) and the orbits surrounding them
- these invariant manifolds play a key role in what follows

Three-Body Problem

- Equations of motion:

$$\boxed{\ddot{x} - 2\dot{y} = -U_x^{\text{eff}}, \qquad \ddot{y} + 2\dot{x} = -U_y^{\text{eff}}}$$

  where

$$U^{\text{eff}} = -\frac{(x^2 + y^2)}{2} - \frac{1-\mu}{r_1} - \frac{\mu}{r_2}.$$

- Have a first integral, the Hamiltonian energy, given by

$$E(x, y, \dot{x}, \dot{y}) = \frac{1}{2}(\dot{x}^2 + \dot{y}^2) + U^{\text{eff}}(x, y).$$

Three-Body Problem

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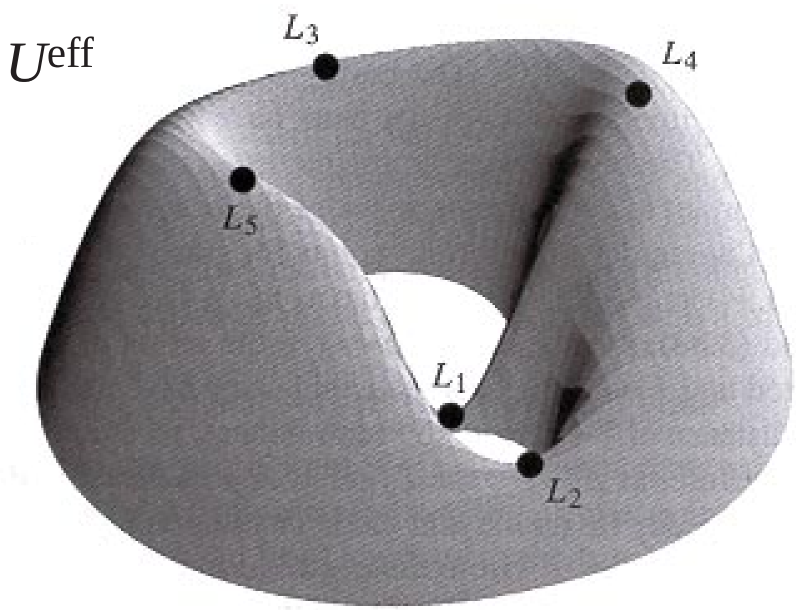
- Energy manifolds are 3-dimensional surfaces foliating the 4-dimensional phase space.
- This is for the planar problem, but the spatial problem is similar.

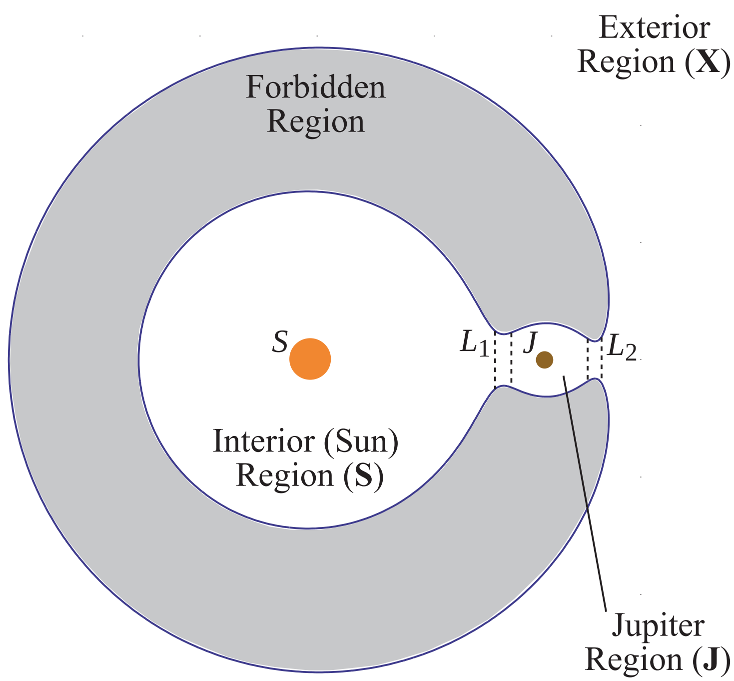
Regions of Possible Motion

Effective potential

- In a rotating frame, the equations of motion describe a particle moving in an effective potential plus a magnetic field (goes back to work of Jacobi, Hill, etc).



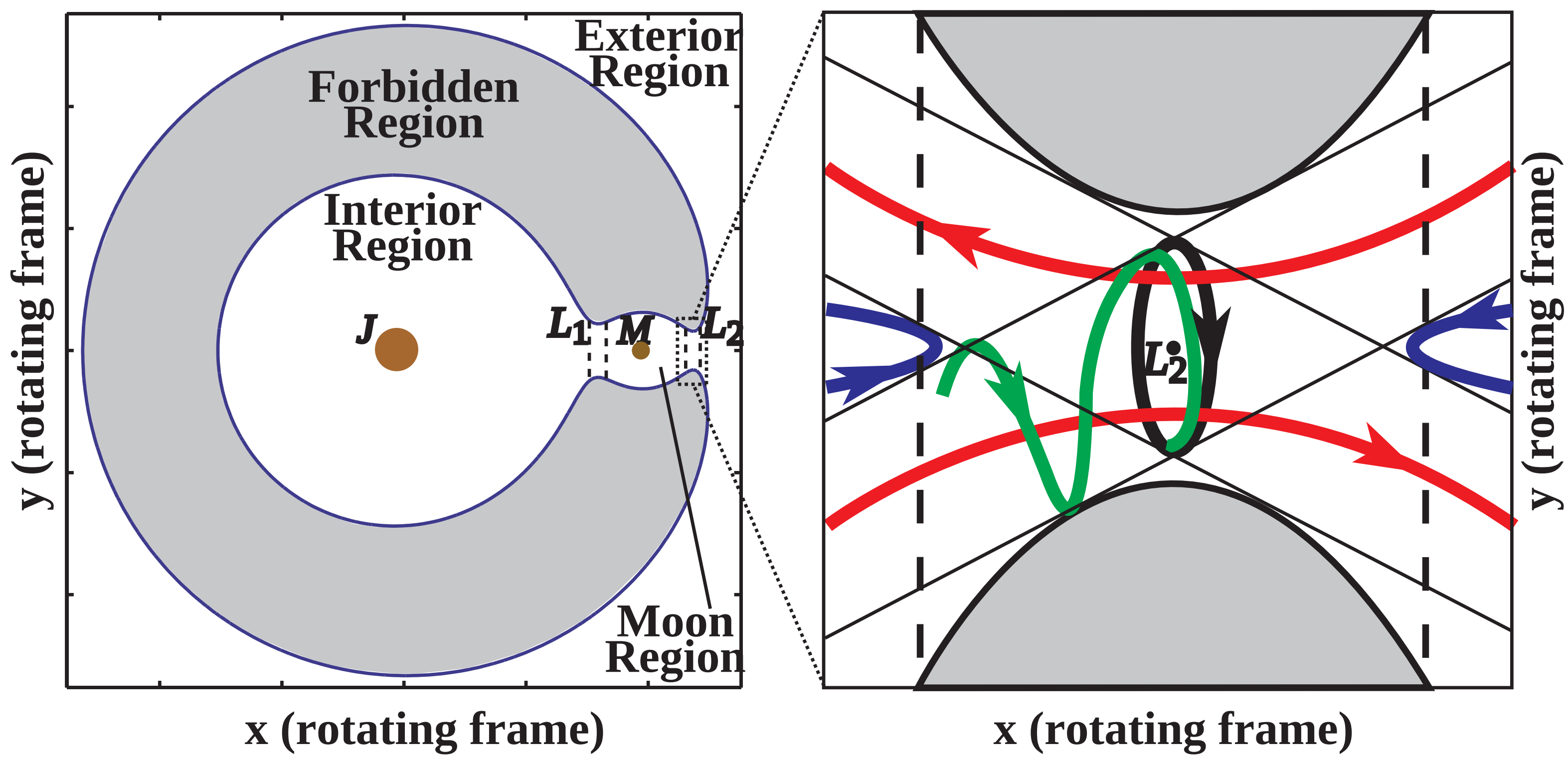
Effective potential



Level set shows accessible regions

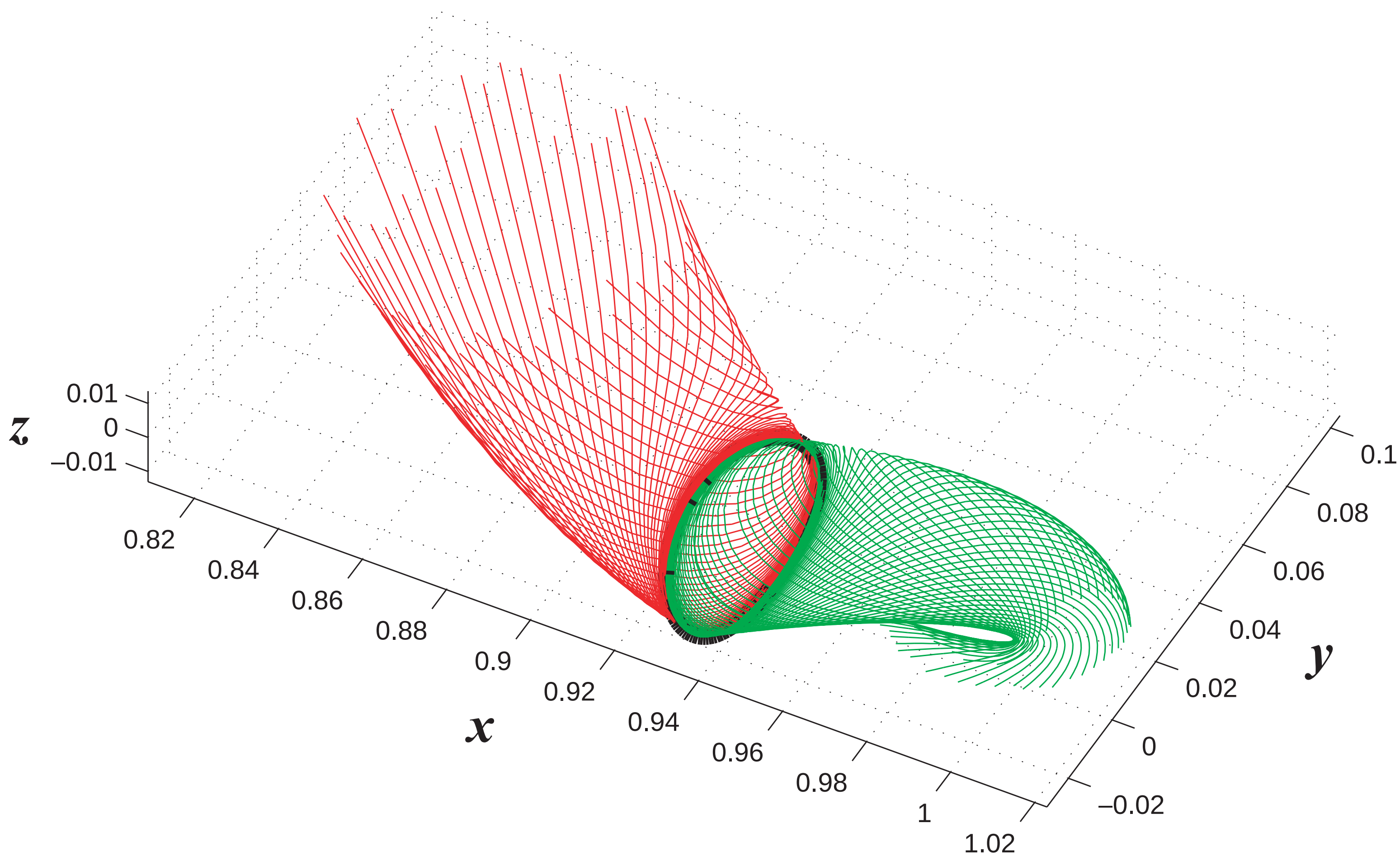
Transport Between Regions

- Dynamics near equilibrium point in spatial problem: **saddle** × **center** × **center**.
  - **bounded orbits** (periodic/quasi-periodic): S^3 (3-sphere)
  - **asymptotic orbits** to 3-sphere: $S^3 \times I$ (**"tubes"**)
  - **transit** and **non-transit** orbits.



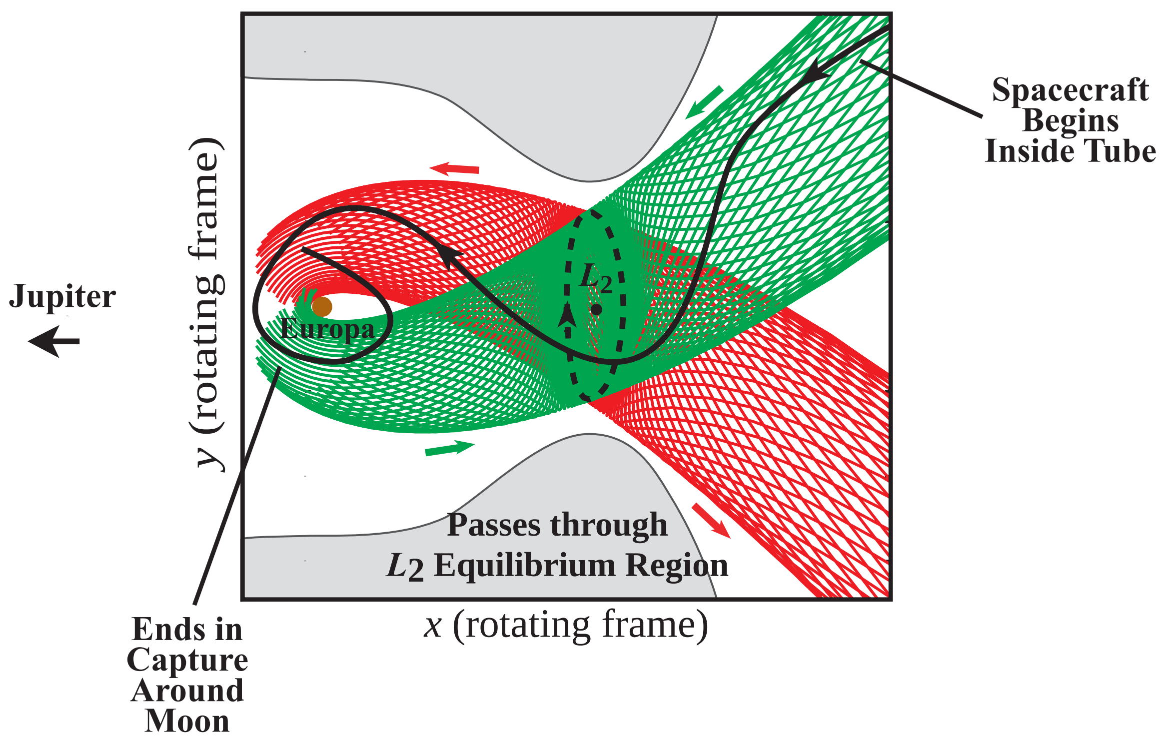
Transport Between Regions

- Asymptotic orbits form **4D invariant manifold tubes** $(S^3 \times I)$ in 5D energy surface.
- red = unstable, green = stable



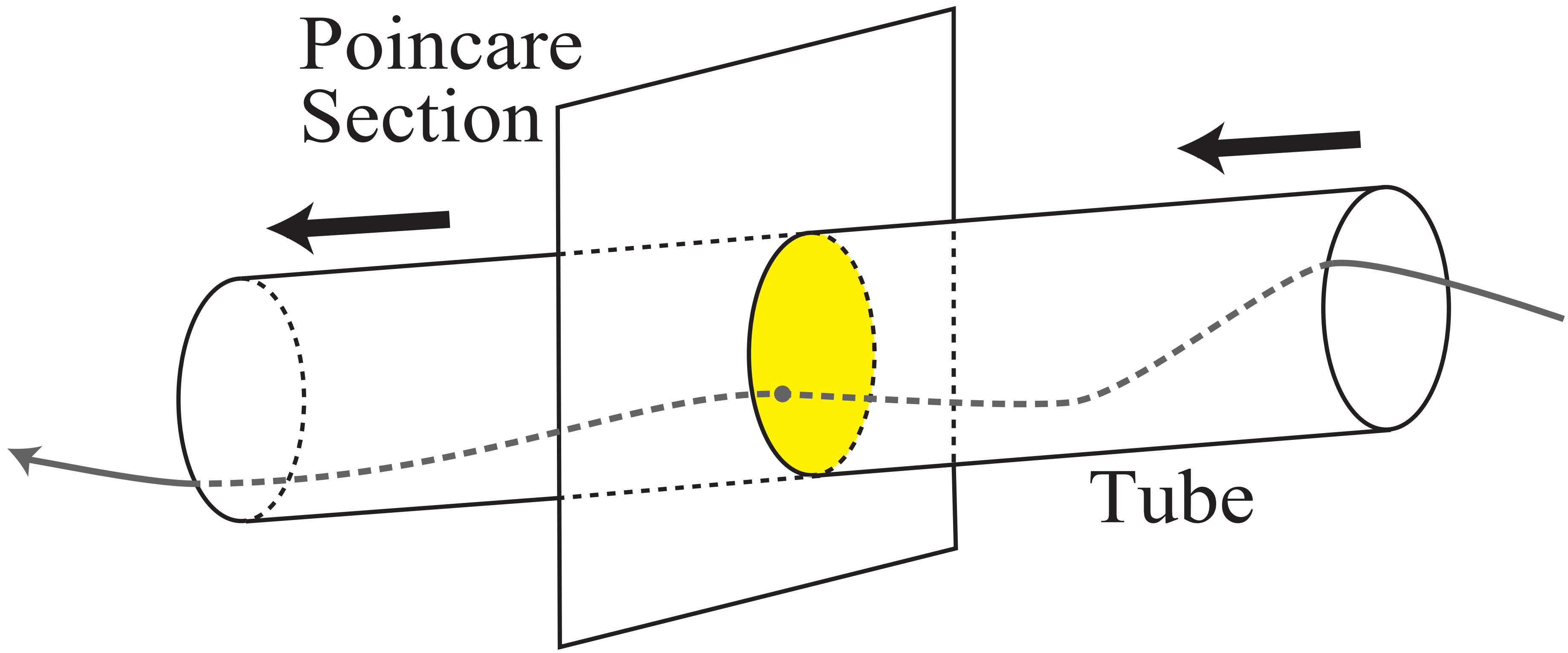
Transport Between Regions

- These manifold tubes play an important role in governing what orbits approach or depart from a moon (**transit orbits**)
- and orbits which do not (**non-transit orbits**)
- transit possible for objects "inside" the tube, otherwise no transit — this is important for transport issues



Transport Between Regions

- Transit orbits can be found using a **Poincaré section** transversal to a tube.

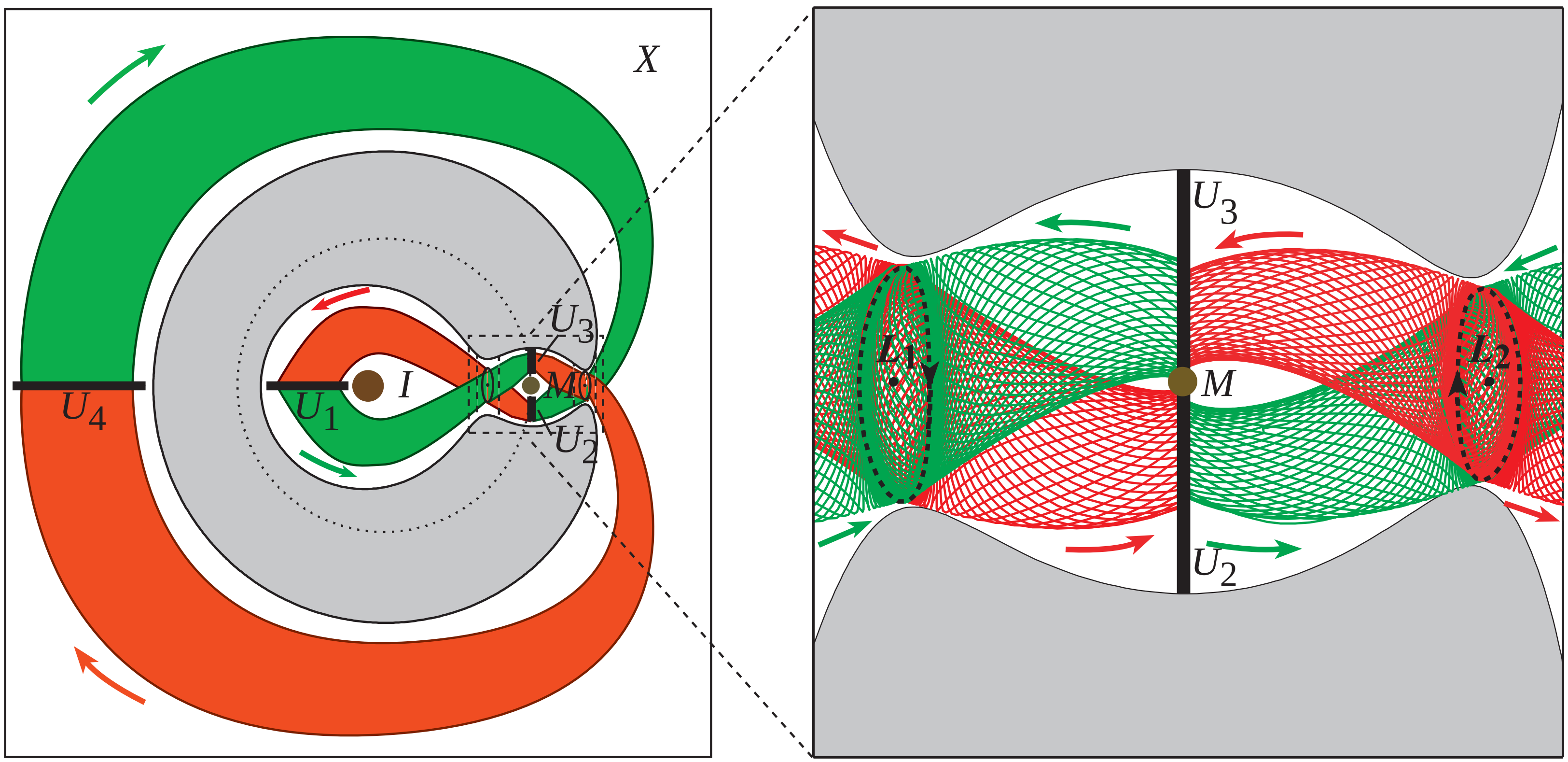


Construction of Trajectories

- One can systematically construct new trajectories, which use little fuel.
  - by linking stable and unstable manifold tubes in the right order
  - and using Poincaré sections to find trajectories "inside" the tubes
- One can construct trajectories involving multiple 3-body systems.

Construction of Trajectories

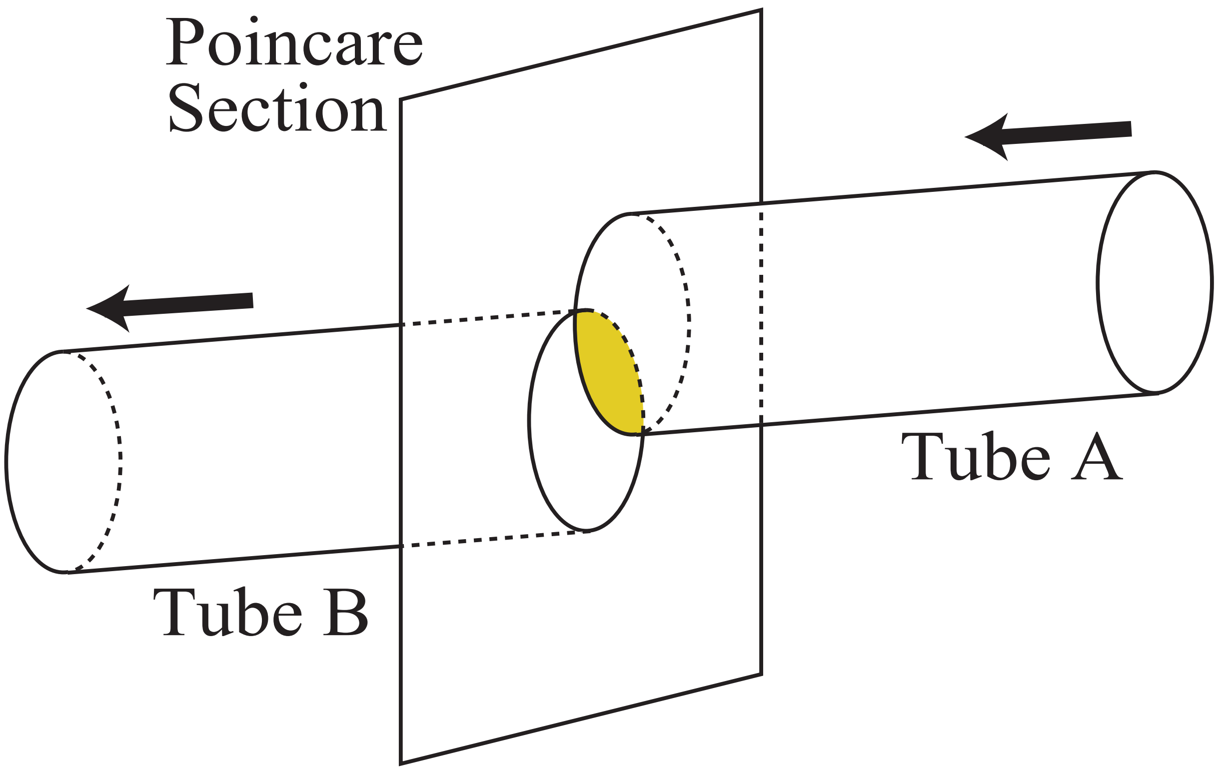
- For a single 3-body system, we wish to **link** invariant manifold tubes to construct an orbit with a **desired itinerary**
- Construction of $(X; M, I)$ orbit.



The tubes connecting the X,M, and I regions.

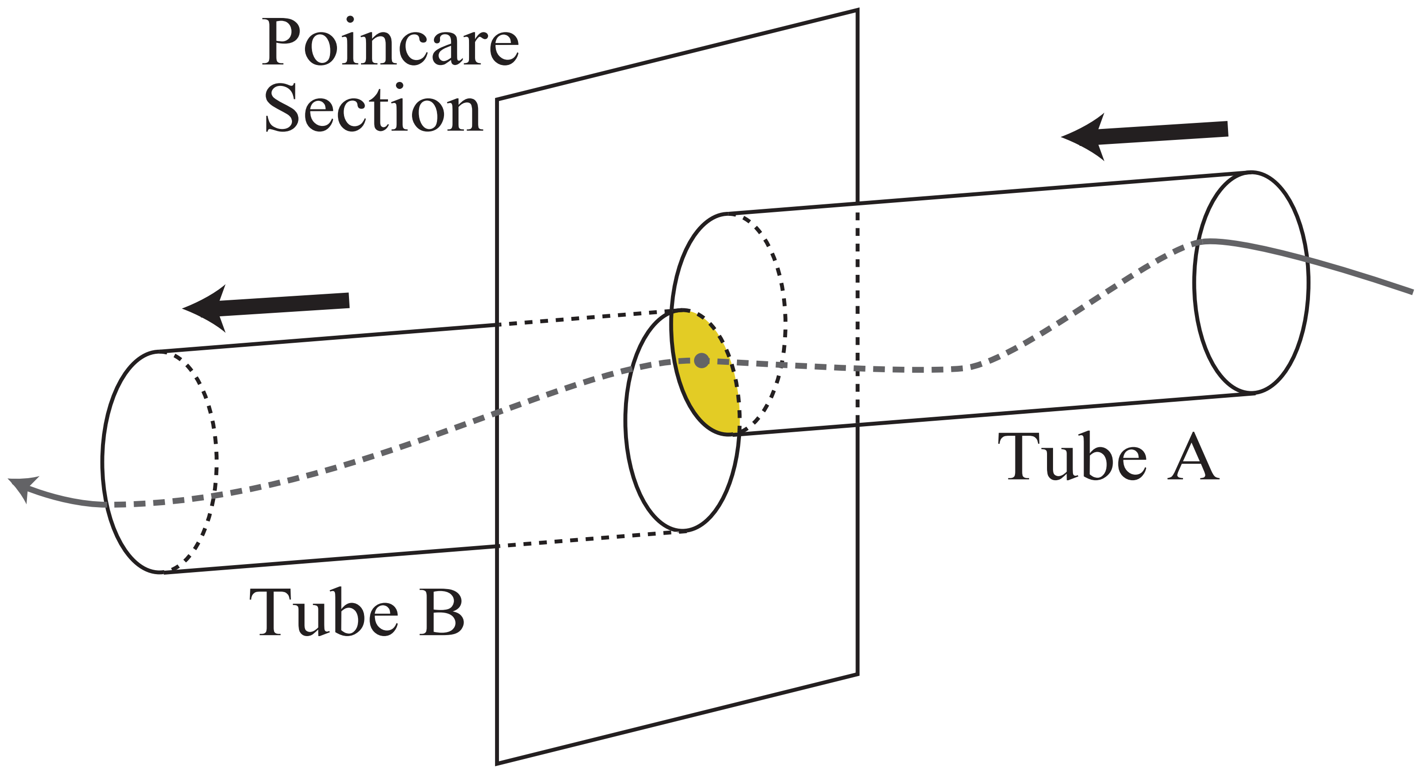
Construction of Trajectories

- First, integrate two tubes until they pierce a common **Poincaré section** transversal to both tubes.



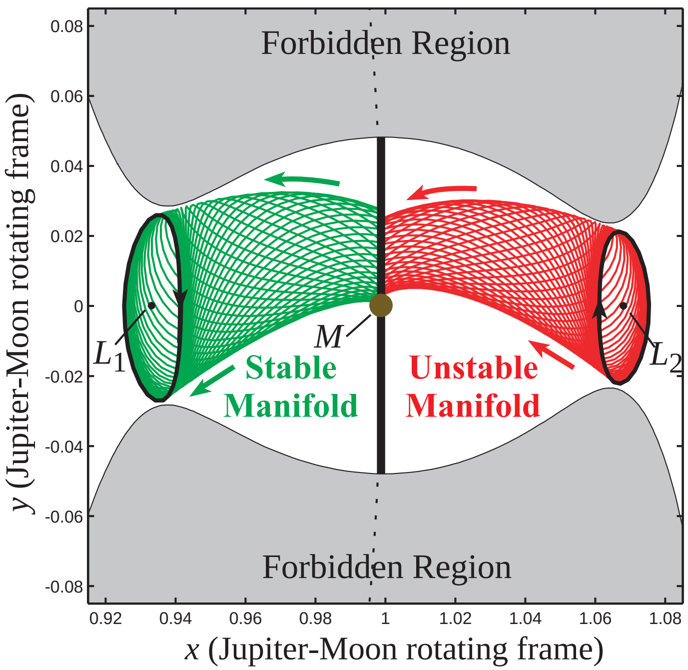
Construction of Trajectories

- Second, pick a point in the region of intersection and integrate it forward and backward.

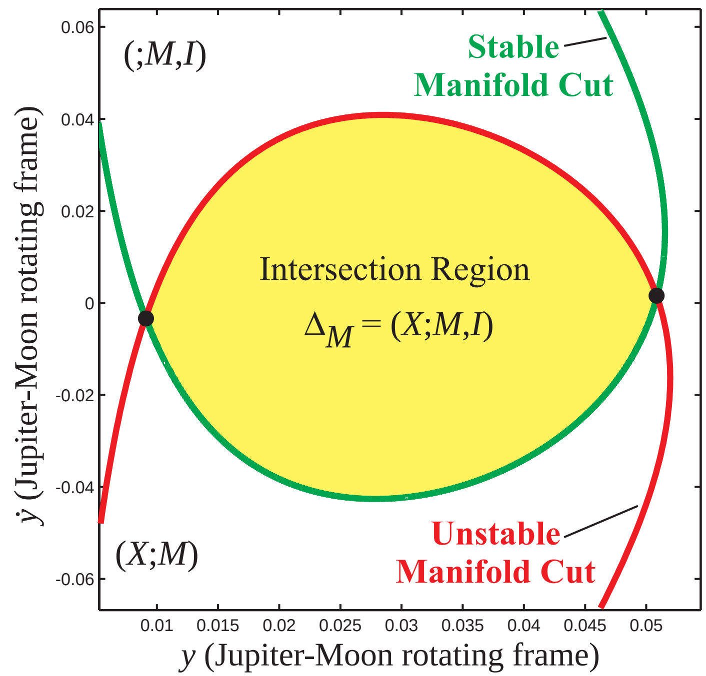


Construction of Trajectories

- **Planar**: tubes $(S \times I)$ separate transit/non-transit orbits.
- **Red curve (S^1)** (Poincaré cut of L_2 **unstable manifold**.
  Green curve (S^1) (Poincaré cut of L_1 **stable manifold**.
- Any point inside the intersection region Δ_M is a $(X; M, I)$ orbit.



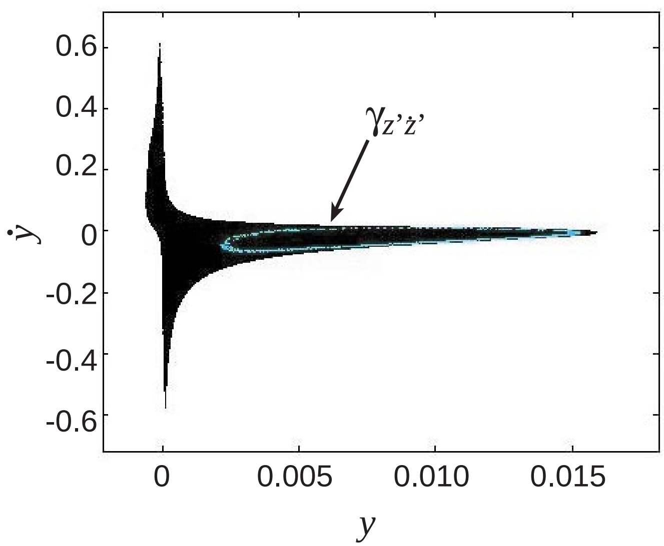
Tubes intersect in position



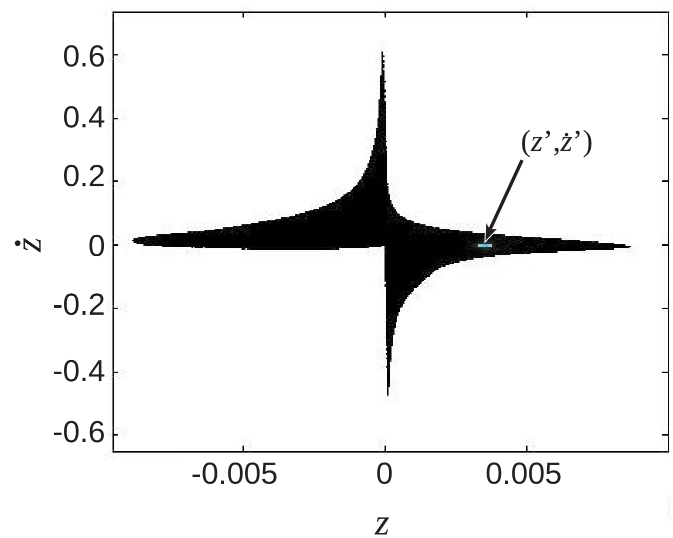
Poincaré section of intersection

Construction of Trajectories

- **Spatial**: Invariant manifold tubes ($S^3 \times I$)
- Poincaré **cut** is a topological **3-sphere** S^3 in $\mathbb{R}^4$.
  - S^3 looks like **disk** $\times$ **disk**: $\xi^2 + \dot{\xi}^2 + \eta^2 + \dot{\eta}^2 = r^2 = r_\xi^2 + r_\eta^2$
- If $z = c, \dot{z} = 0$, its **projection** on $(y, \dot{y})$ **plane** is a **curve**.
- For **unstable** manifold: any point inside this **curve** is a $(X; M)$ orbit.



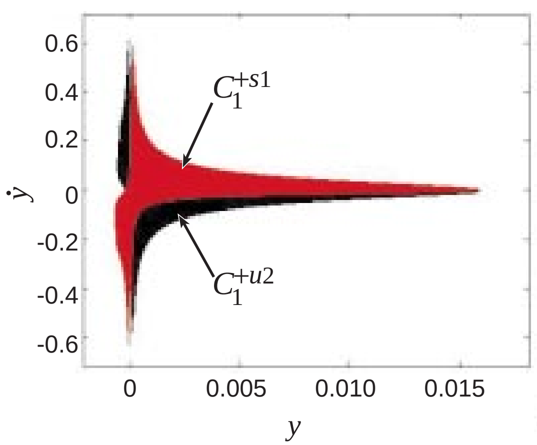
$(y, \dot{y})$ Plane



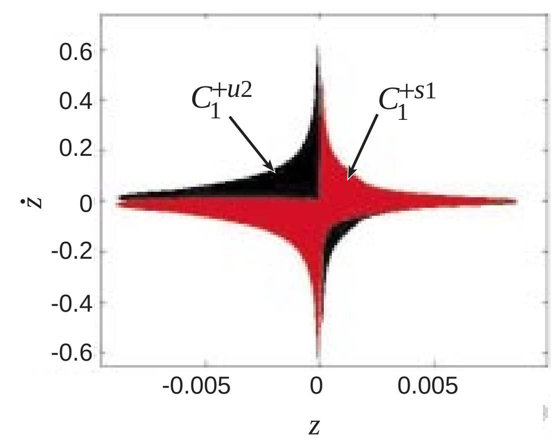
$(z, \dot{z})$ Plane

Construction of Trajectories

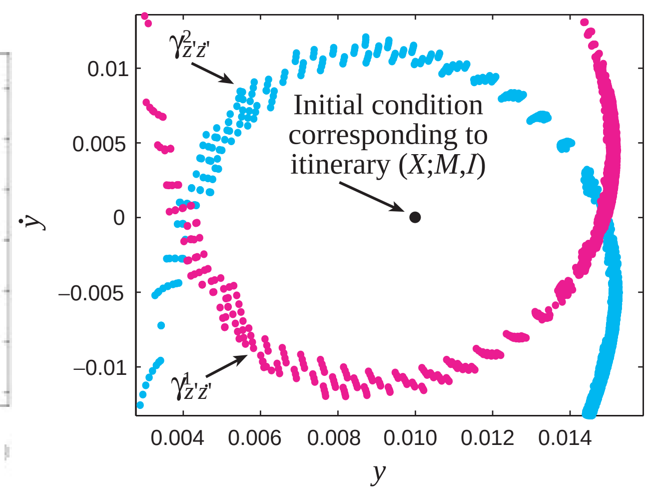
- Similarly, while the cut of the **stable** manifold tube is S^3, its projection on $(y, \dot{y})$ plane is a **curve** for $z = c, \dot{z} = 0$.
- Any point inside this **curve** is a (M, I) orbit.
- Hence, any point inside the **intersection region** Δ_M is a $(X; M, I)$ orbit.



$(y, \dot{y})$ Plane

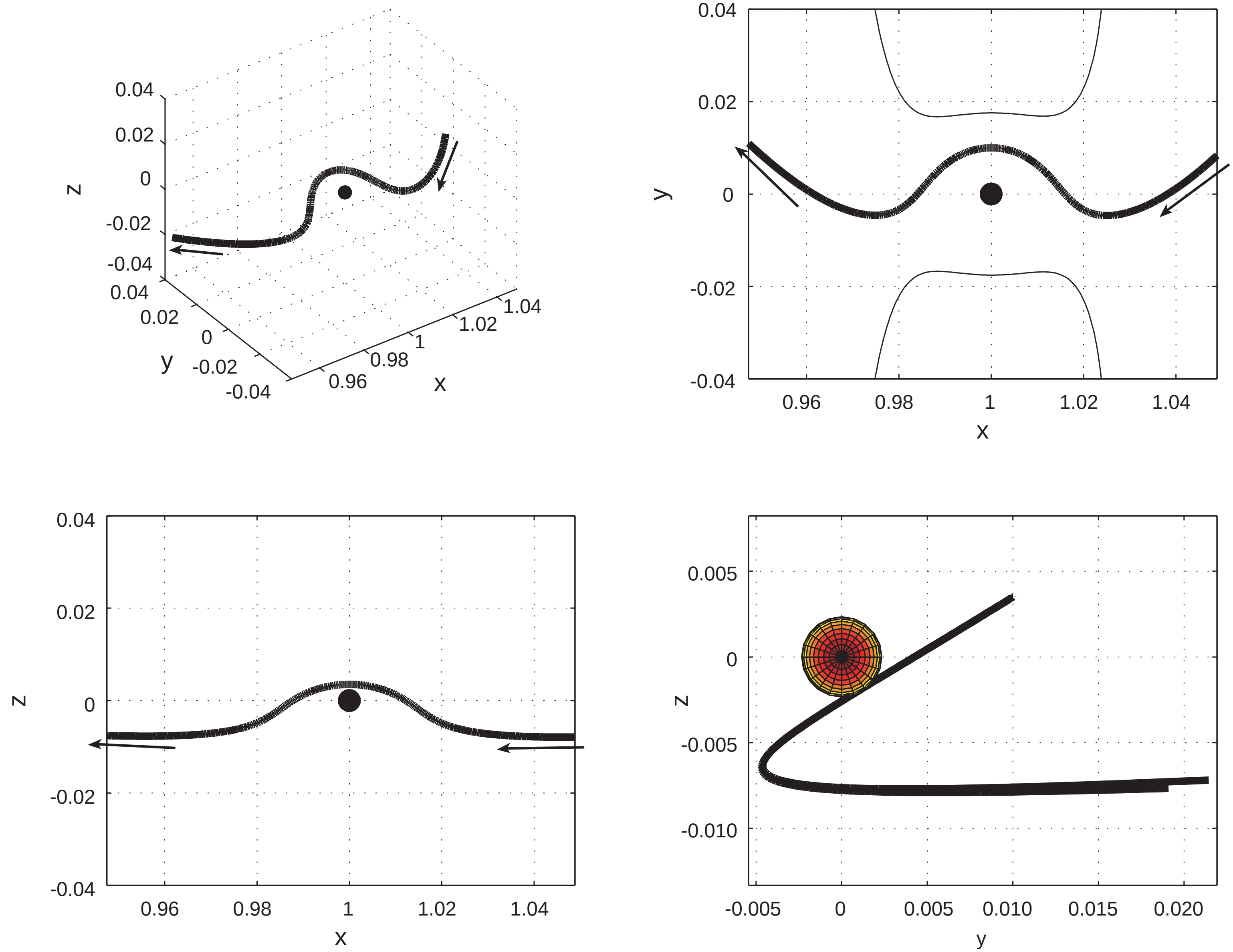


$(z, \dot{z})$ Plane



Intersection Region

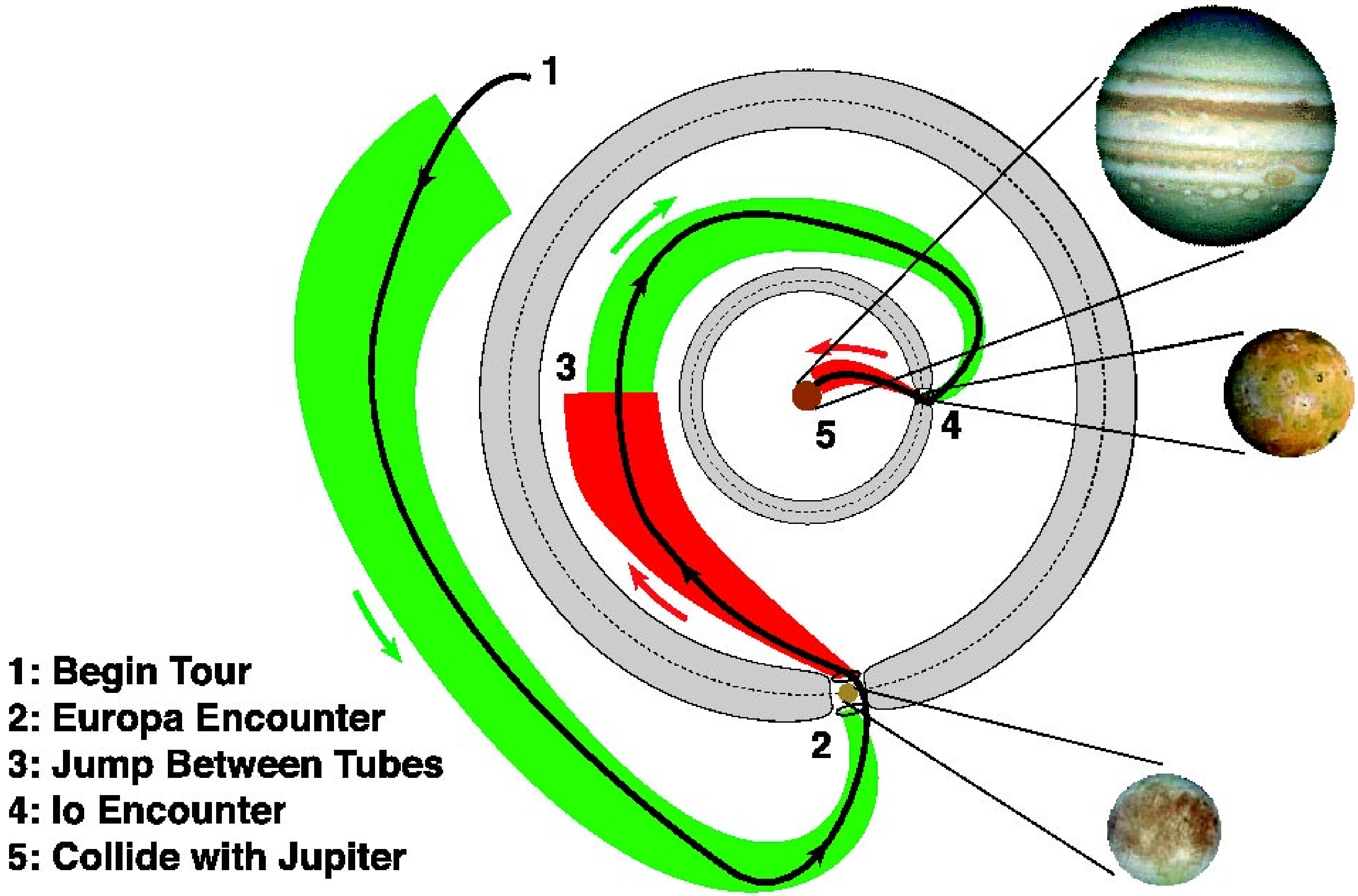
Construction of Trajectories



Construction of an (X, M, I) orbit

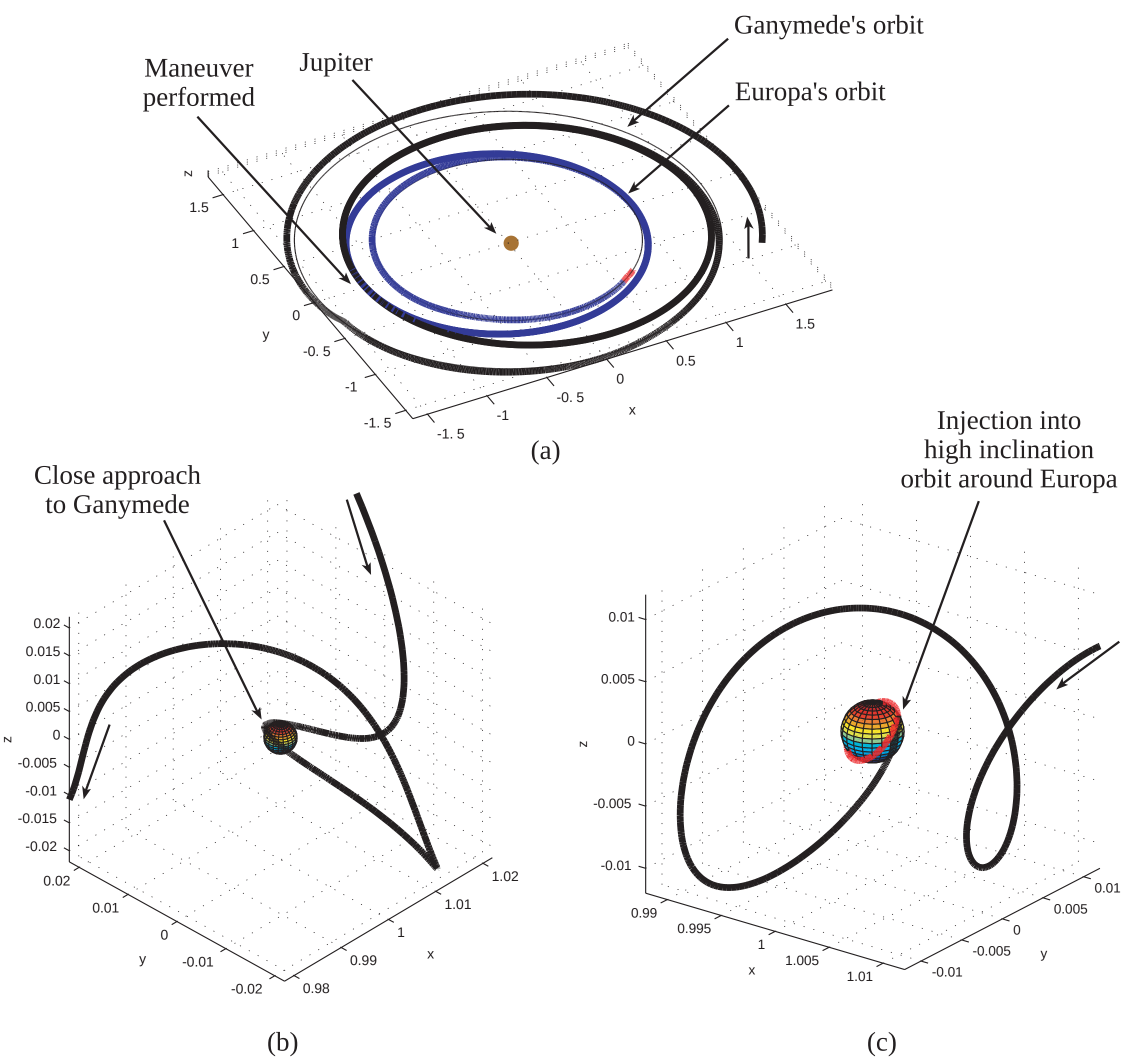
Tour of Jupiter's Moons

- ***Tours of planetary satellite systems.***
  - *Example 1:* Europa → Io → Jupiter



Tour of Jupiter's Moons

- *Example 2:* Ganymede → Europa → injection into Europa orbit



(a)

(b)

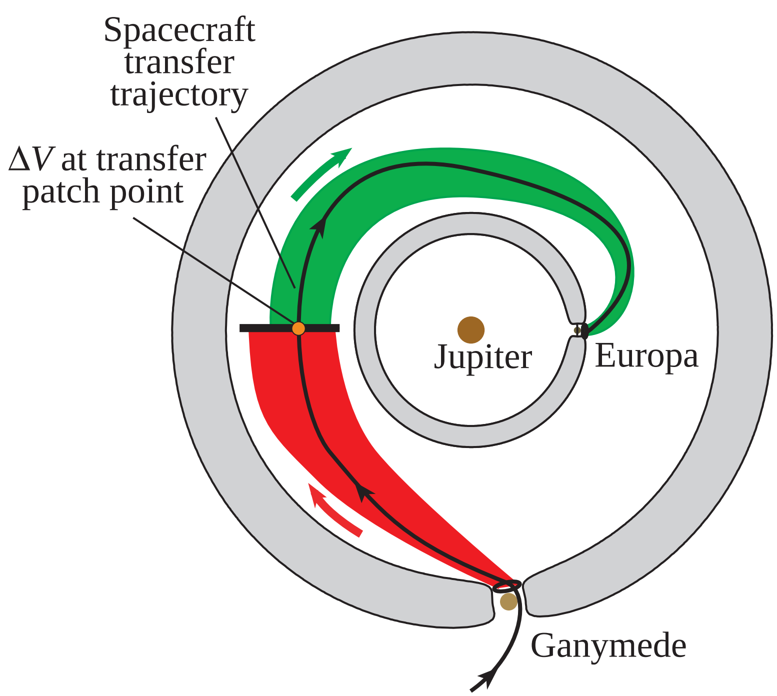
(c)

Tour of Jupiter's Moons

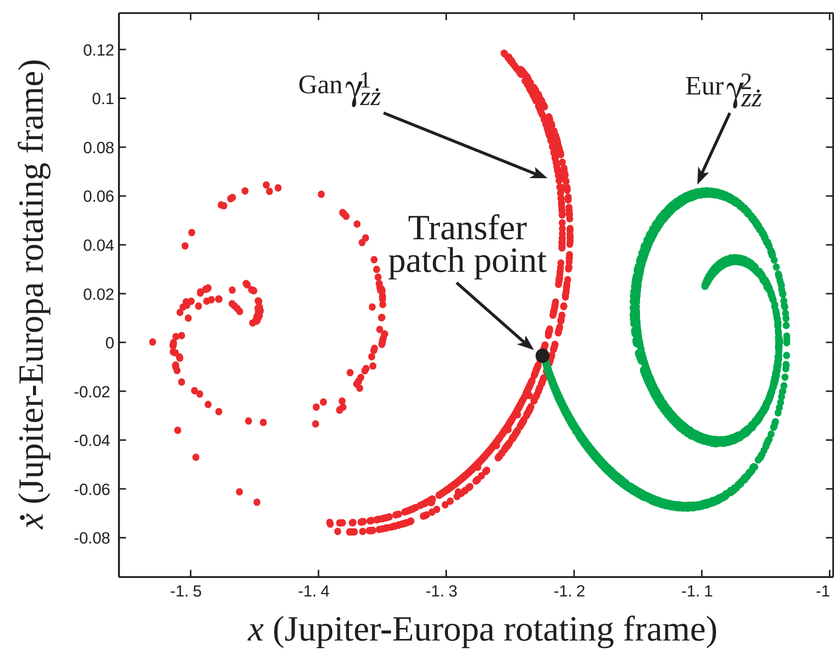
pgt-3d-orbit-eu.qt

Tour of Jupiter's Moons

- The **Petit Grand Tour** can be constructed as follows:
  - Approximate 4-body system as 2 nested **3-body systems**.
  - Choose an appropriate Poinaré section.
  - Link the invariant manifold tubes in the proper order.
  - Integrate initial condition (patch point) in the 4-body model.



Look for intersection of tubes



Poincaré section at intersection

Further Work

- *More refinement needed*

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- ***More refinement needed***
  - Control over inclination of final orbit

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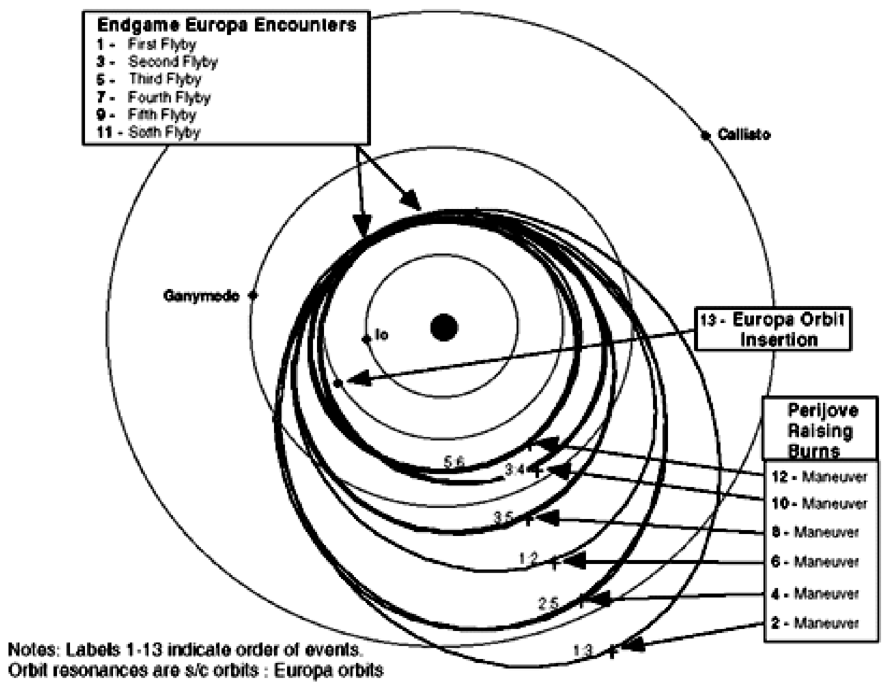
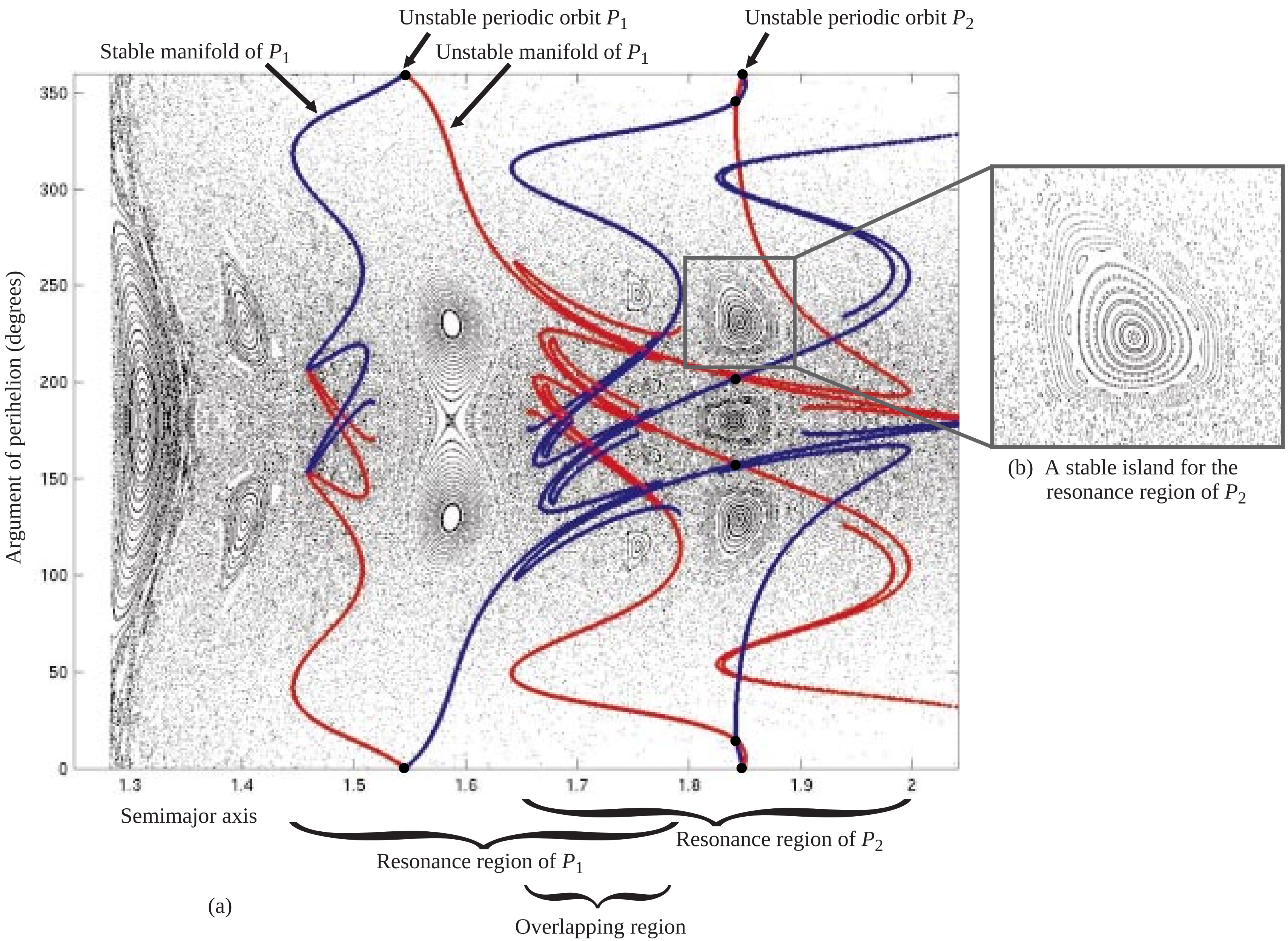
  - Control over inclination of final orbit
  - Further reduce fuel cost using other techniques
    - Resonance transition to pump down orbit via repeated close approaches to the moons



Using resonance transition to pump down orbit

Further Work



Resonance transition as shown by Poincaré map

Further Work

- Use low (continuous) thrust, rather than impulsive
- Combine with optimal control software (e.g., NTG, COOPT)

References

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