Optimal flapping strokes for self-propulsion in a perfect fluid

Shane Ross

Engineering Science & Mechanics, Virginia Tech

www.esm.vt.edu/∼sdross

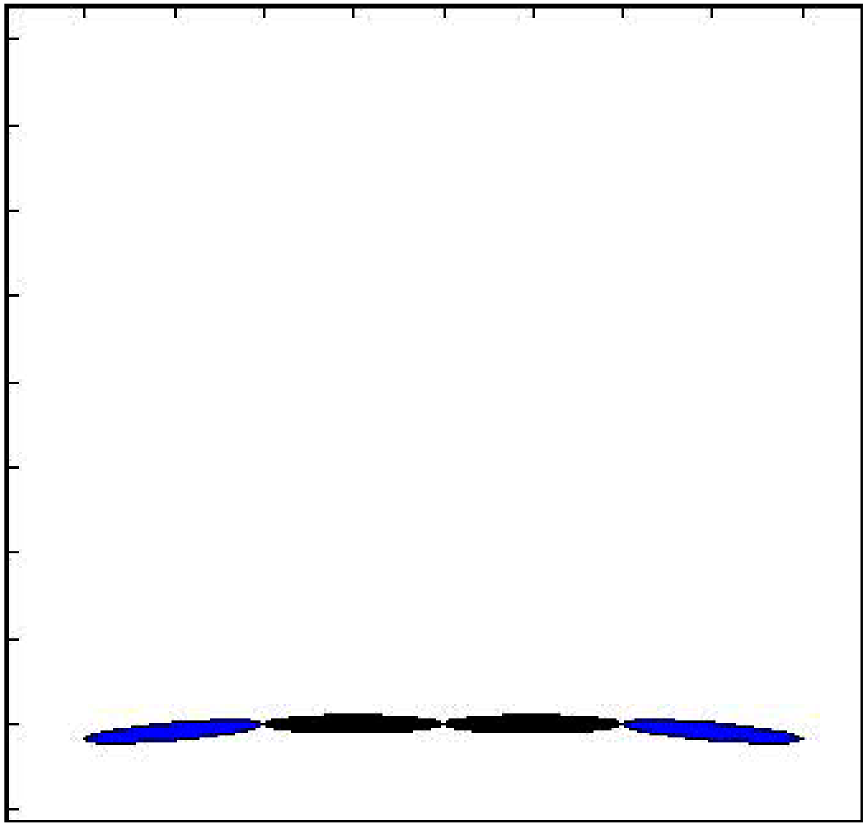
ACC 2006

In this talk

- Design and control of vehicles with articulated bodies
  - Jointed four-link model of self-propulsion via large shape changes
  - Geometric structure of propulsive shape change strokes
  - Efficient strokes: mechanical structure preserving optimal control code

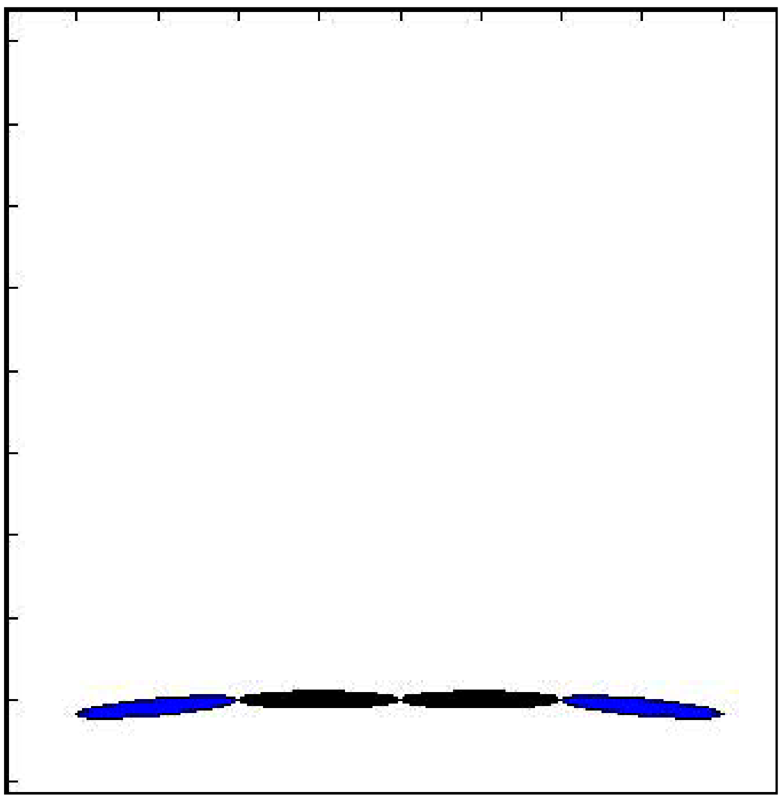
Locomotion model

- Symmetrical four-link model propelling from rest
  - vortex shedding not solely responsible for locomotion, as noted by Saffman [1967]
  - applies methods used previously on three-link carangiform fish (Kanso et al. [2005])

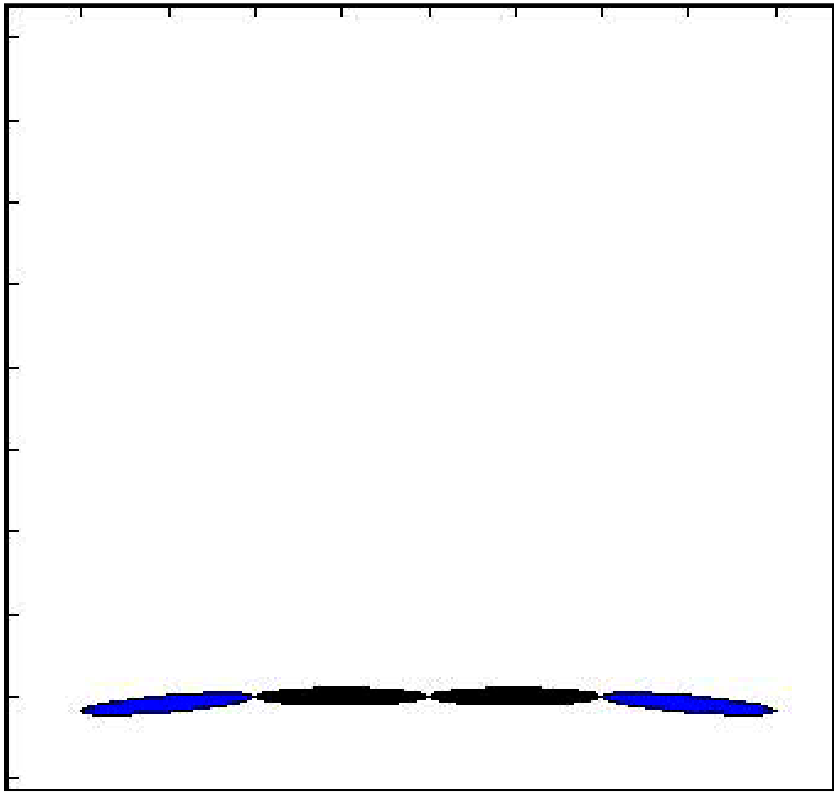


Locomotion model

- example of "holonomy drive"
  - seen in, e.g., self-propulsion of microorganisms at low Reynolds number
  - **locomotion** based on sequence of shapes, not relative speed of shape change
  - but, **control effort** is based on relative speed of shape change

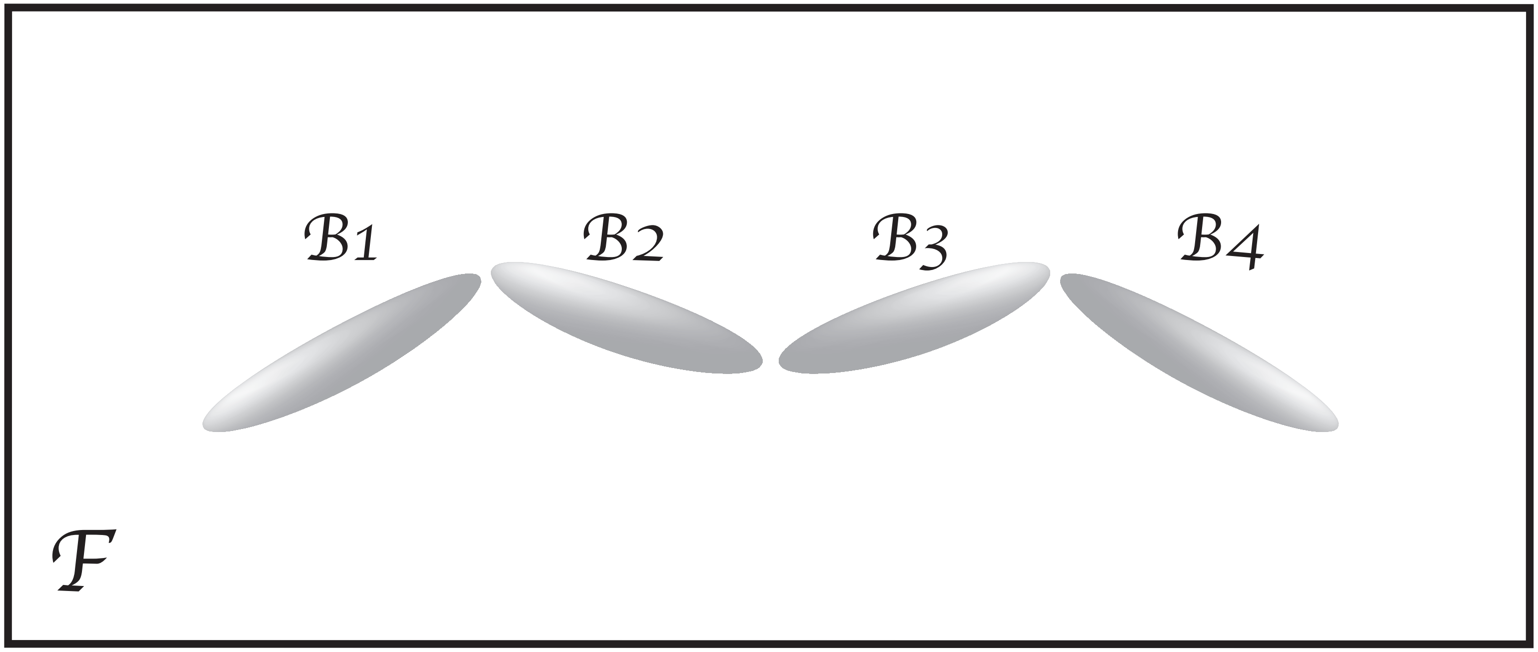


more efficient



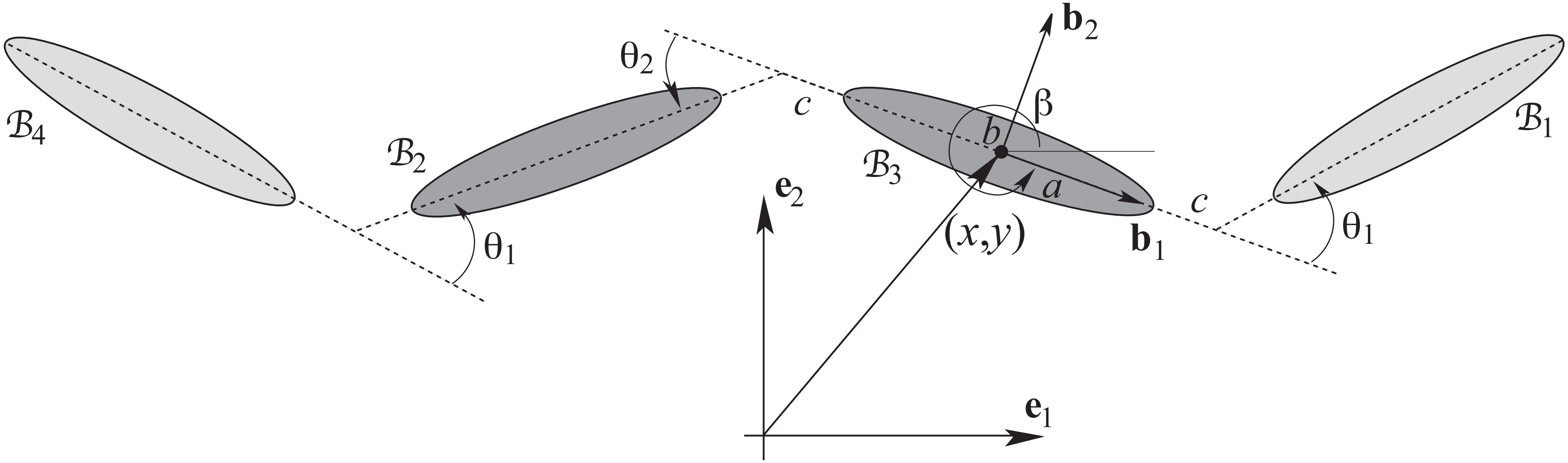
less efficient

Four-link flapper model



- $\mathcal{F}$ is assumed to be inviscid, incompressible and irrotational for all time
- Potential flow ($\mathrm{u} = \nabla\phi, \nabla^2 u = 0$) with slip across solid boundaries
- Articulated body of four 4 rigid links $\mathcal{B}_i$ connected by hinge joints
- Bilaterally symmetric "flapping": four links is minimum necessary for locomotion in potential flow; allows for non-reciprocal shape changes

Four-link flapper model



- Neutrally buoyant identical links
- Links: slender ellipsoidal geometry with axes a, b, where $b/a \ll 1$
- Joints: equipped with muscles which generate torques to achieve a desired stroke
- $g = (\beta, x, y)$, orientation, position of $\mathcal{B}_3$ w.r.t. $\{\mathrm{e}_1, \mathrm{e}_2\}$ – **net locomotion variables**
- $\theta = (\theta_1, \theta_2)$, orientation of $\mathcal{B}_1$, $\mathcal{B}_2$, and $\mathcal{B}_4$ relative to $\mathcal{B}_3$ – **shape space variables**

Solid-fluid Lagrangian $L = T_s + T_f$

- Lagrangian = solid + fluid kinetic energy,

$$L = \sum_{i=1}^{4} T_{\mathcal{B}_i} + T_f = \frac{1}{2} \sum_{i=1}^{4} \xi_i^T \, \mathbb{I} \, \xi_i,$$

  with $\xi_i = (\Omega_i, v_i)^T$ the velocity of the link $\mathcal{B}_i$ w.r.t. the $\mathcal{B}_i$-fixed frame and $\mathbb{I}_{ij} = \mathbb{I}$, including the added inertia, is the same for all links.

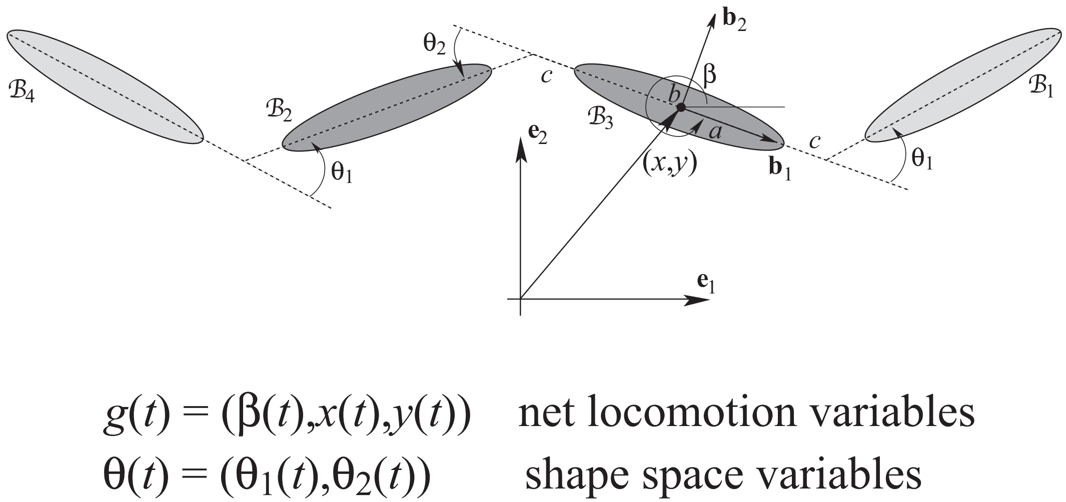
- The Lagrange-d'Alembert variational principle yields the forced Euler-Lagrange equations:

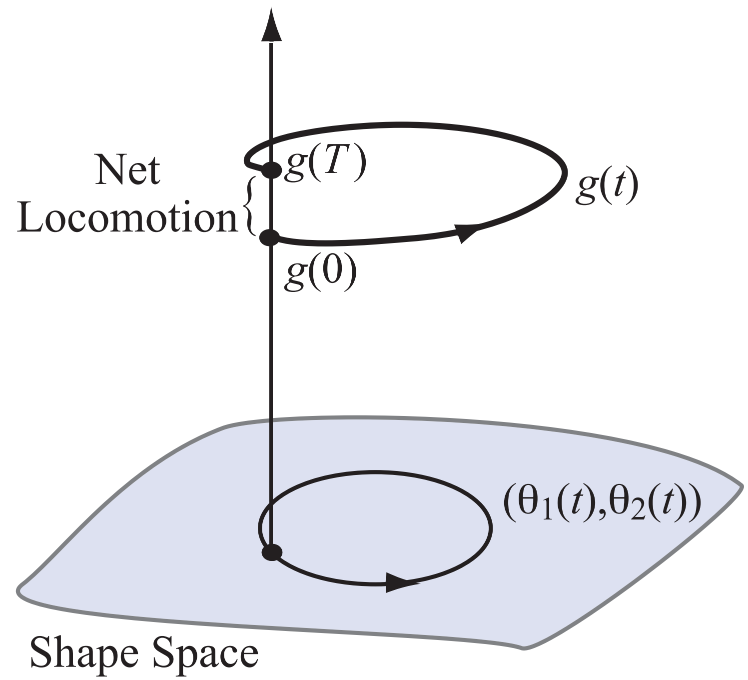
$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{g}_i}\right) - \frac{\partial L}{\partial g_i} = 0, \qquad i = 1, 2, 3, \tag{1}$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\theta}_i}\right) - \frac{\partial L}{\partial \theta_i} = \tau_i, \qquad i = 1, 2, \tag{2}$$

  where the internal torques $\tau(t)$ are exerted by actuators (or muscles) associated with the joints.

Geometric mechanics description





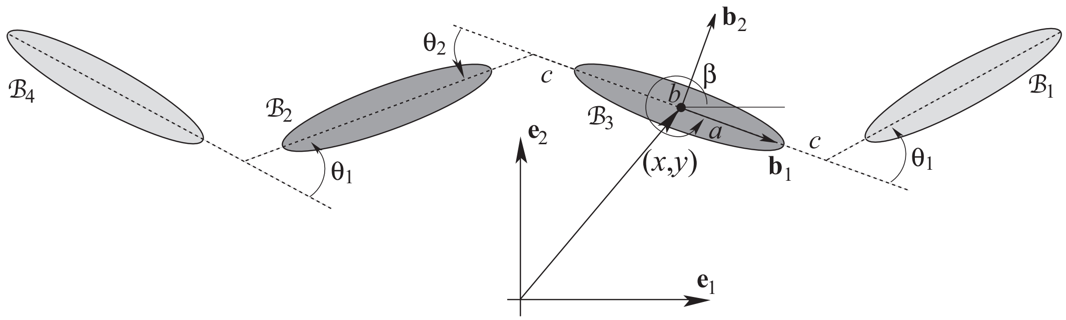
- When the motion starts from rest, (1) yields

$$g^{-1}\dot{g} = -\mathcal{A}(\theta)\dot{\theta}, \qquad (3)$$

where $g \in SE(2)$, the group of rotations and translations in $\mathbb{R}^2$.

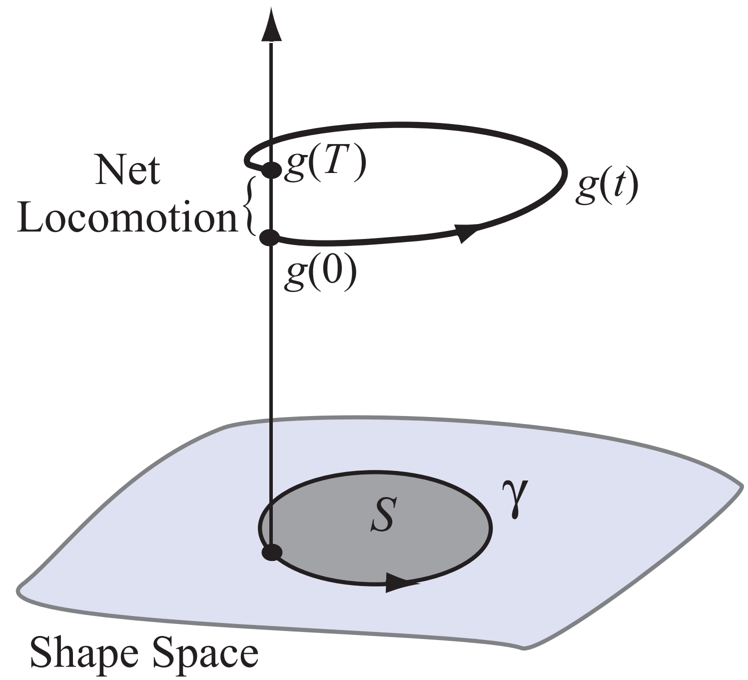
- Given a curve $\theta(t) = \big(\theta_1(t), \theta_2(t)\big), t \in [0, T]$, we solve (3) for $g(t) = \big(\beta(t), x(t), y(t)\big)$ and solve (2) for the torques $\tau(t)$

Stroke: closed loop in shape space



$g(t) = (\beta(t),x(t),y(t))$ net locomotion variables

$\theta(t) = (\theta_1(t),\theta_2(t))$ shape space variables



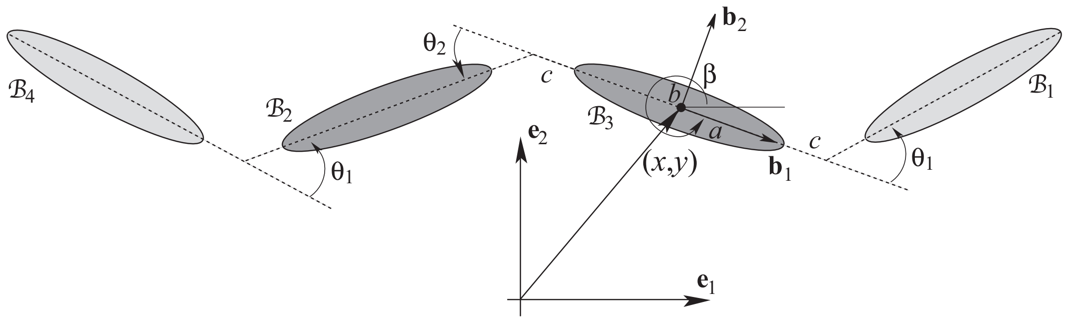
- A **stroke**: if $\theta(t)$ traces out a **closed loop** γ in shape space Θ from time 0 to T,

$$g(T) = g(0) \exp\left(-\int_S d\mathcal{A}(\theta)\right).$$

  where S is the region of Θ whose boundary is the loop γ

- Note: independent of time parametrization of curve γ

Net locomotion from one stroke



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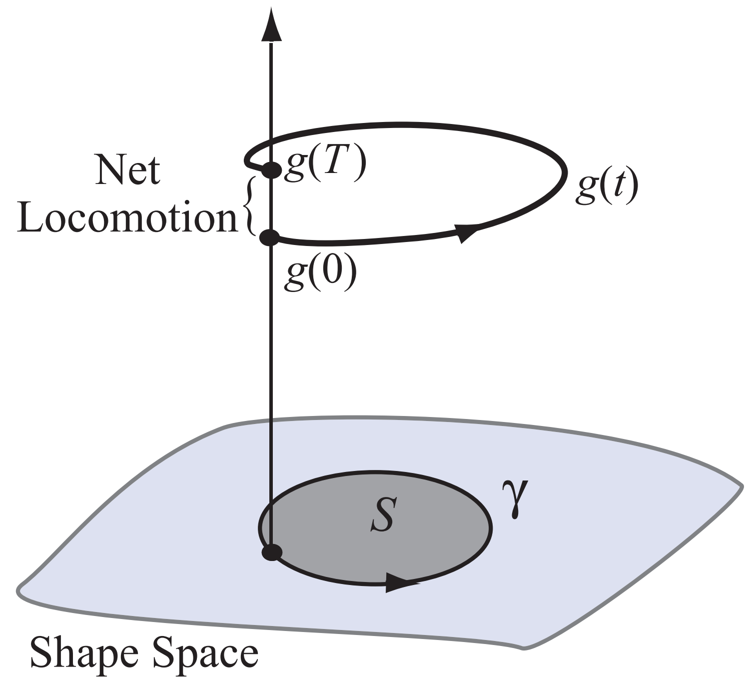
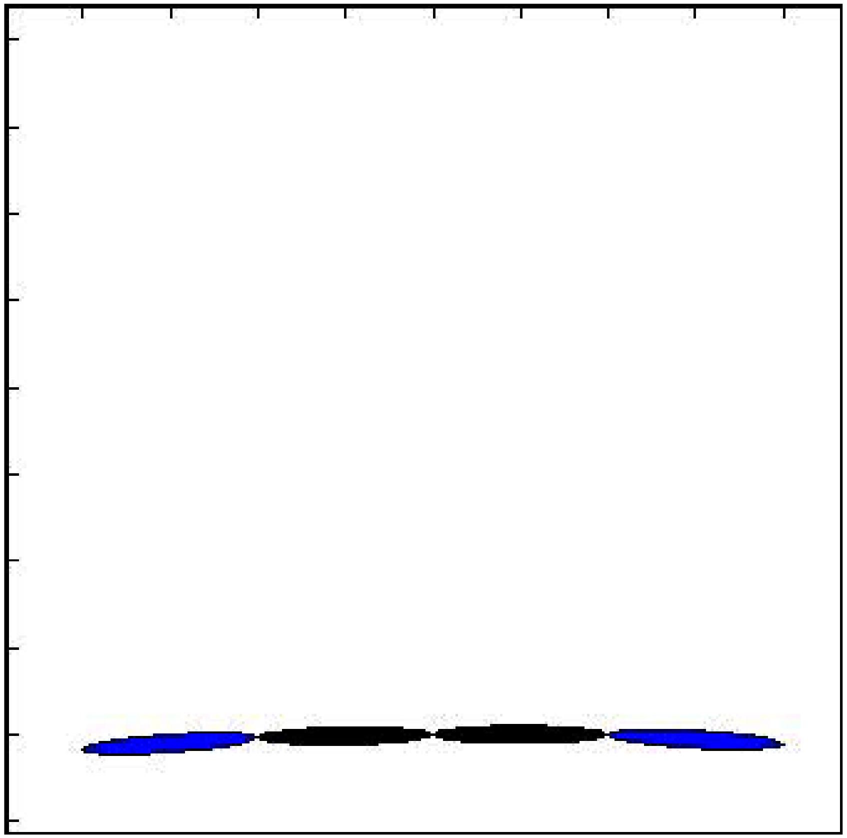
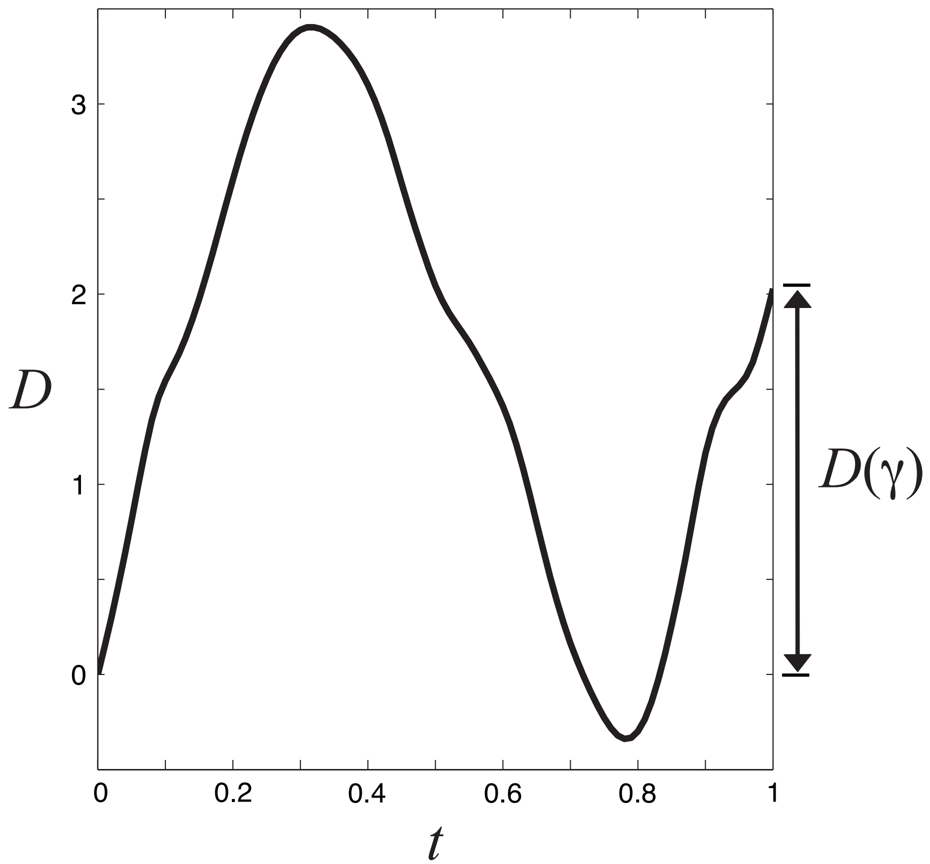


- i.e., **net locomotion** achieved, $g(T) - g(0)$, is a function of the loop geometry only (not on the instantaneous speeds along which the loop is traversed)

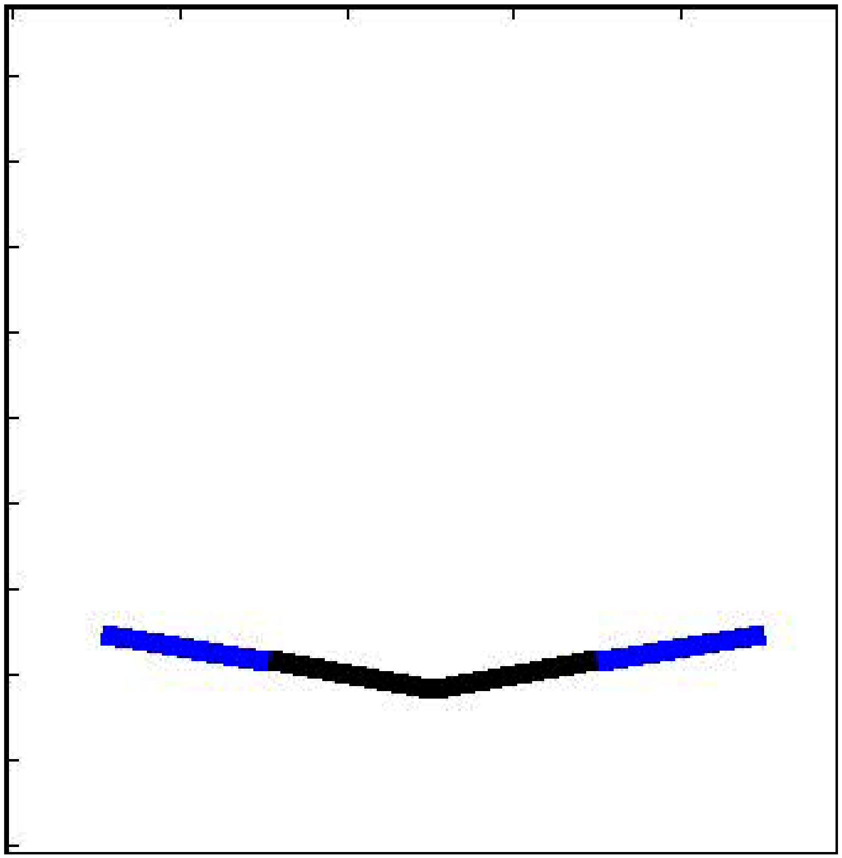
Net locomotion from one stroke

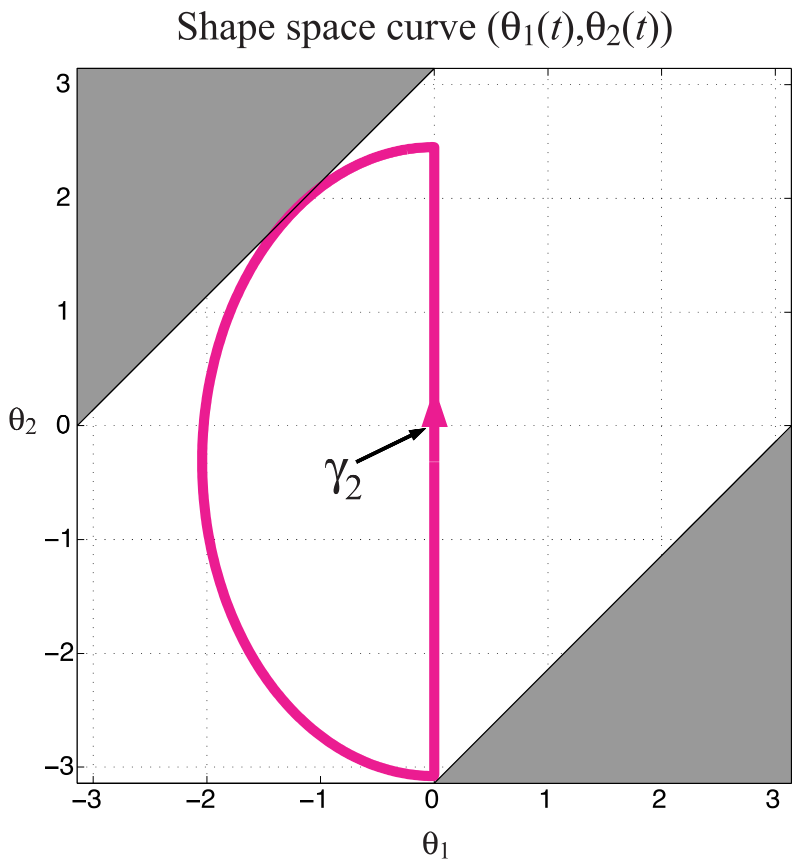




- When a flapper has completed one stroke, it is back to its original shape, but has translated a distance $D(\gamma)$
- $D(\gamma) = D([\gamma])$ where $[\gamma]$ = equivalence class of strokes w.r.t. time reparametrization

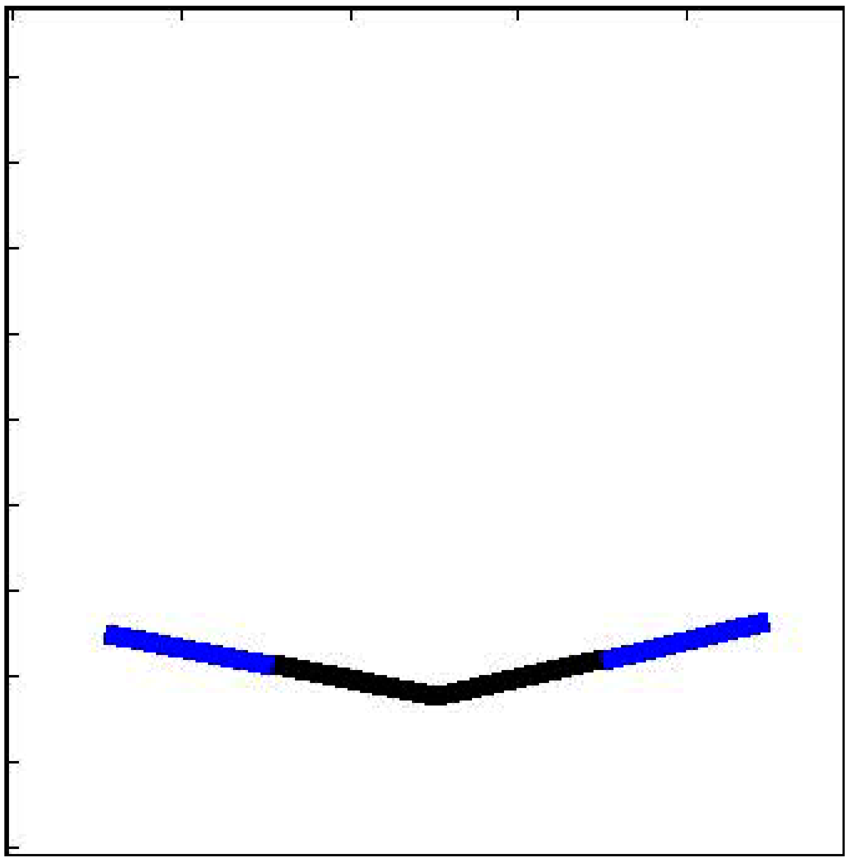
Example stroke loops γ

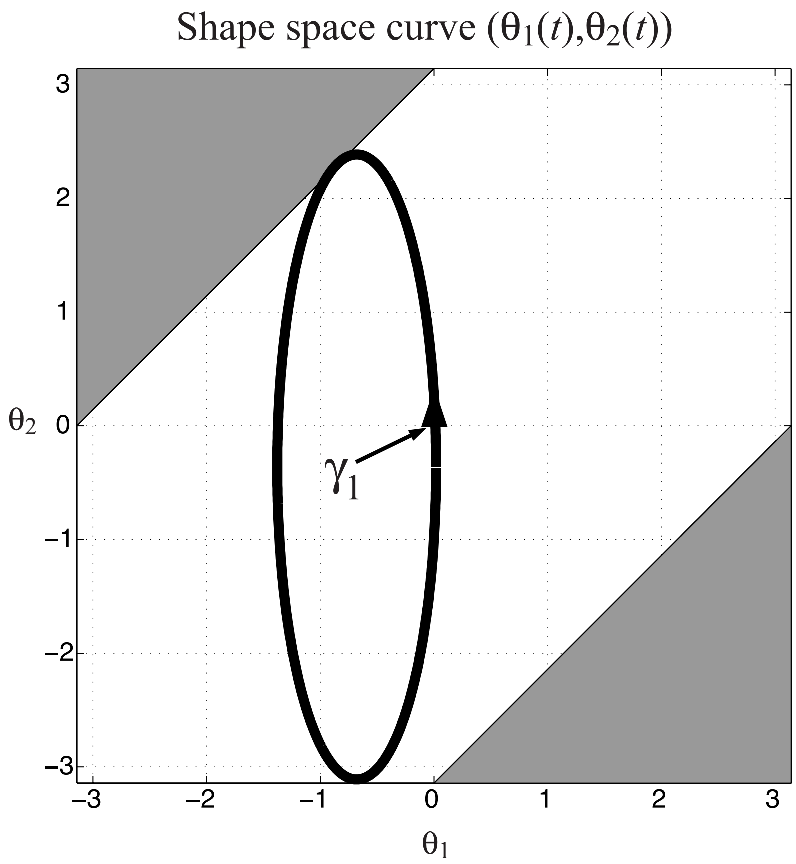




- simple expressions for closed curves $(\theta_1(t), \theta_2(t))$

Example stroke loops γ





- simple expressions for closed curves $(\theta_1(t), \theta_2(t))$

Optimal stroke loops

- Optimal strokes minimize the (torque) control effort per unit distance travelled

$$\delta(\gamma) = W(\gamma)/D([\gamma])$$

where

$$W(\gamma) \;=\; \int_0^T |\tau|^2 \, dt,$$

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- Approximate $q(t) = (g(t), \theta(t))$ by a discrete path q_n at $t_n = \{0, h, 2h, \ldots, Nh\}$

  And approximate control $\tau(t)$ by discrete torques τ_n.

- Use DMOC (Discrete Mechanics & Optimal Control) algorithm of Junge, et al. [2005]
- Based on discretization of Lagrange D'Alembert variational principle
  $\Rightarrow$ discrete forced Euler-Lagrange equations
- Preserves mechanical structure; conserves momentum

Optimization via DMOC

- Search over initial curves γ_{init}
- DMOC algorithm

$\gamma_{\text{init}} \Longrightarrow$

Minimize discrete cost function

$$\delta_d = \sum_n C_d(q_n, \tau_n, t_n) \; = \; \sum_n (\sum_i \tau_{n\,i}^2) h$$

subject to forced Euler-Lagrange equations
(as nonlinear equality constraints)

$$\begin{aligned} D_2 L_d(q_{n-1}, q_n) + D_1 L_d(q_n, q_{n+1}) + \tau_{n-1} + \tau_n &= 0 \\ p_0 + D_1 L_d(q_0, q_1) + \tau_0 &= 0 \\ -p_1 + D_2 L_d(q_{N-1}, q_N) + \tau_{N-1} &= 0 \end{aligned}$$

$\Longrightarrow \gamma_{\text{opt}}$

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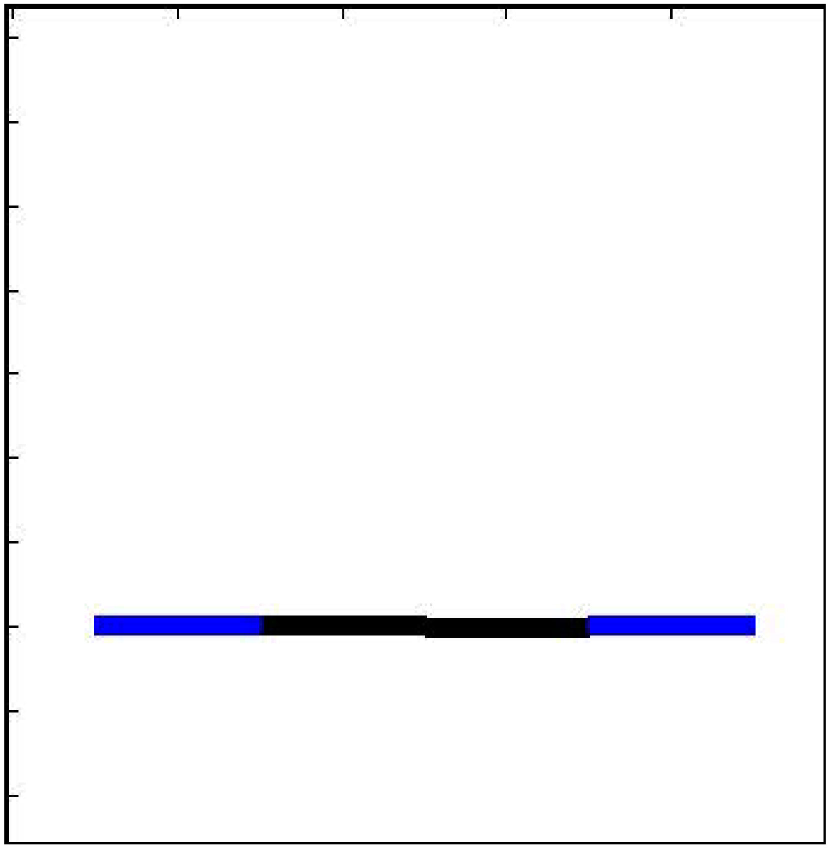
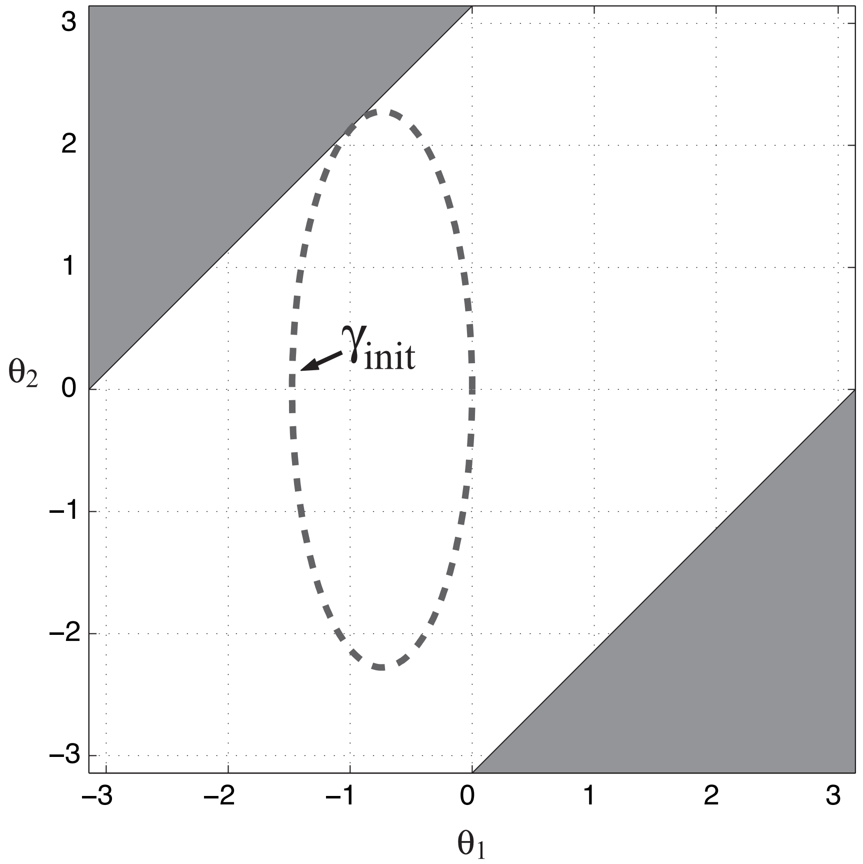
$\Longrightarrow \gamma_{\text{opt}}$

- Implemented using SQP package of Matlab
- With $N = 100$, optimization usually takes a few minutes

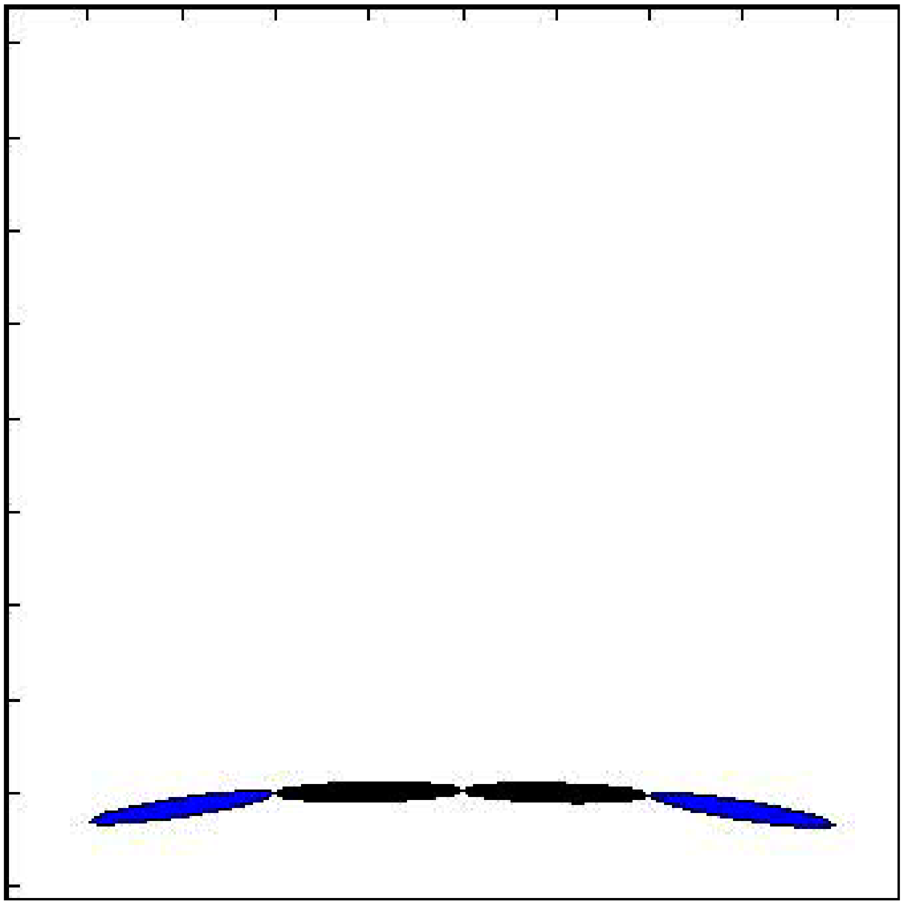
Optimization via DMOC

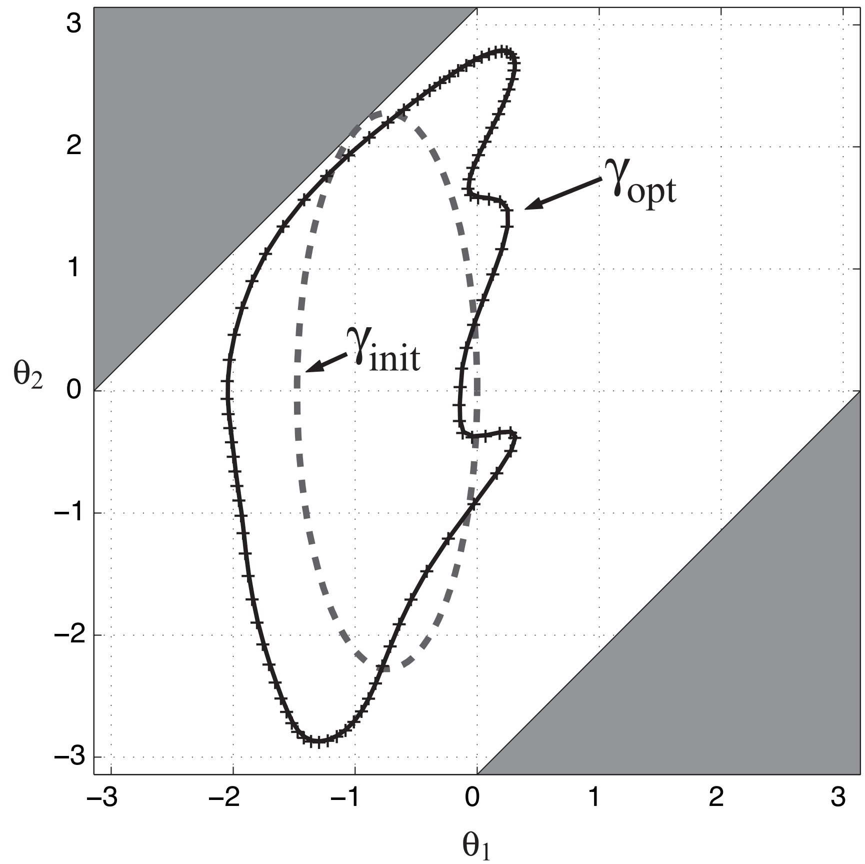




$\gamma_{\text{init}} \Longrightarrow$ Minimize discrete version of $\delta(\gamma)$ subject to discrete forced Euler-Lagrange eqs $\Longrightarrow \gamma_{\text{opt}}$

Optimization via DMOC

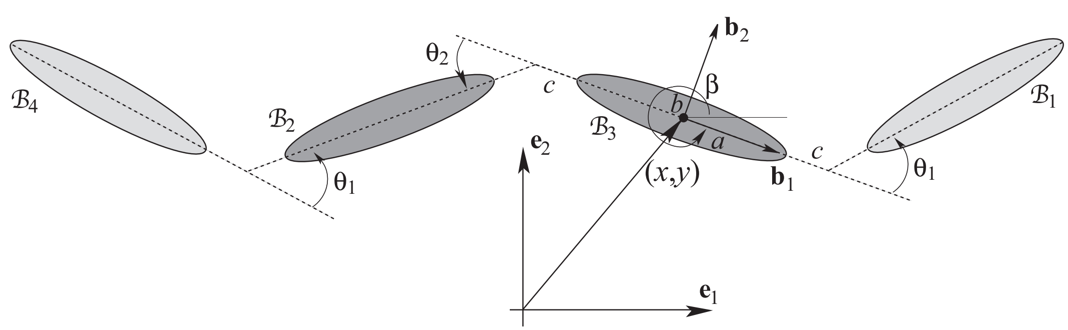




$\gamma_{\text{init}} \Longrightarrow$ Minimize discrete version of $\delta(\gamma)$ subject to discrete forced Euler-Lagrange eqs $\Longrightarrow \gamma_{\text{opt}}$

Summary

- Developed a jointed four-link model of self-propulsion via cyclic strokes in a 2D perfect fluid.



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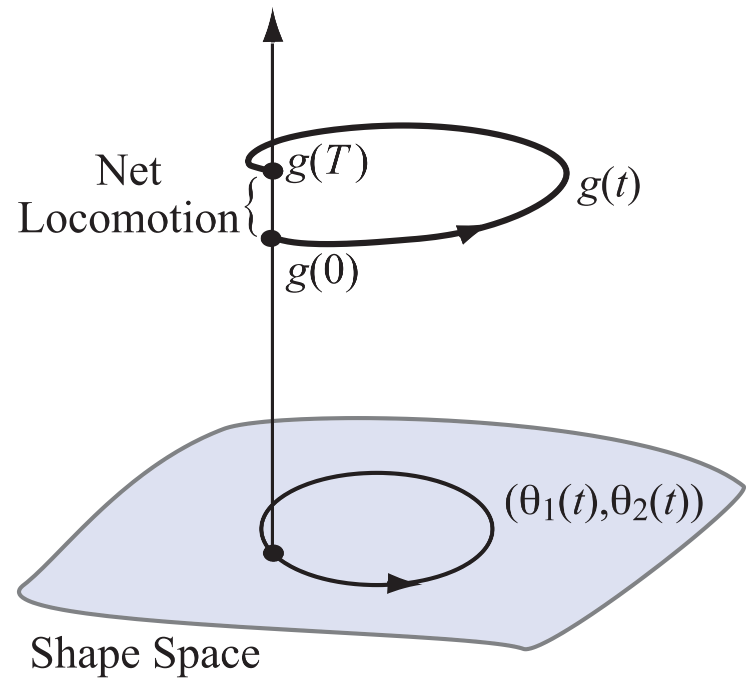

Summary

- Determined which stroke yields the greatest locomotive efficiency, minimizing the control effort (muscle activity) per unit distance traveled.